Lifting Transforms on Graphs and Their Application to Video Coding

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Outline

Motivation

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- **Related Work:**
  - Image Processing:
    - [Velisavljevic et al., 2006, Le Pennec and Mallat, 2005, Shen and Ortega, 2008, Fattal, 2009]
  - Video Coding: Lifting in the temporal domain for video coding [Secker and Taubman, 2003, Pesquet-Popescu and Bottreau, 2001]

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Nevertheless... these transforms filter in the spatial or in the temporal domain independently

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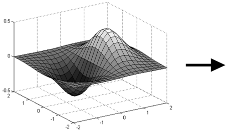
Our new transform is able to filter following **N-dimensional directions** of high correlation... How do we do this?

Introduction to the transform



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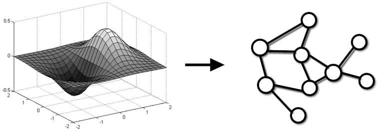

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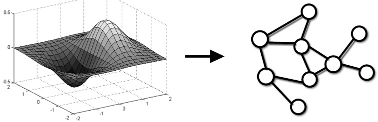

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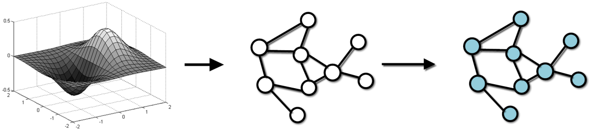

- Describe an N-dimensional signal as a graph
  - Correlated samples should be linked and vice versa
  - Graph captures correlation in N-dimensional domains
- Perform a lifting transform on this graph
  - Filtering operations using neighbor nodes $\Longrightarrow$ graph construction defines filtering directions... we can avoid filtering across large discontinuities!!

Objectives

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- Description of a general framework to construct **N-dimensional directional transforms**
- **Optimization of lifting transforms on graphs**
  - In the sense of energy compaction
- **Video coding application of the proposed transform**

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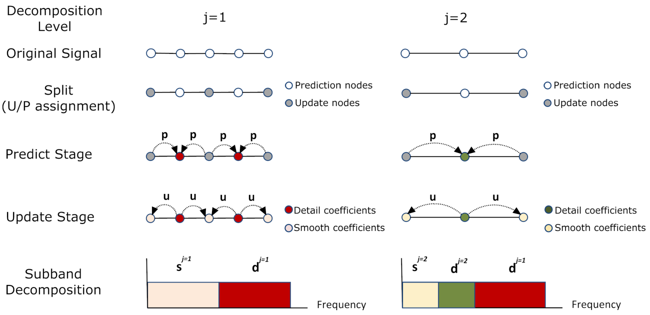

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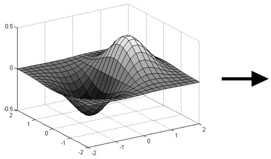

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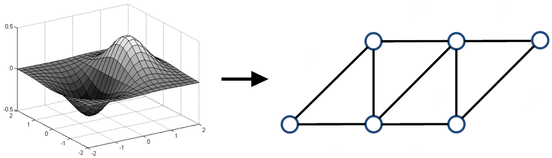

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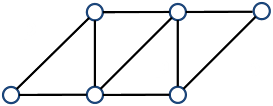

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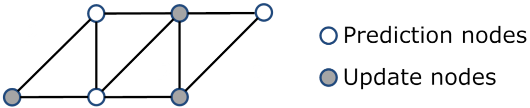

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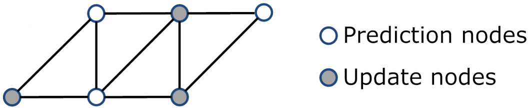

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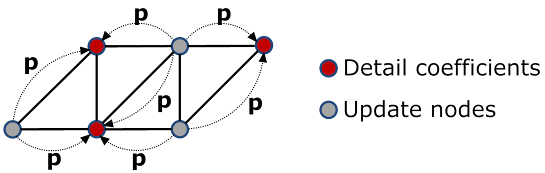

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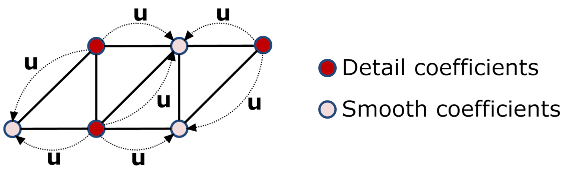

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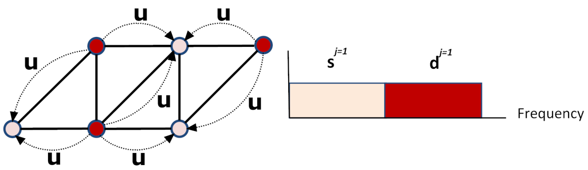

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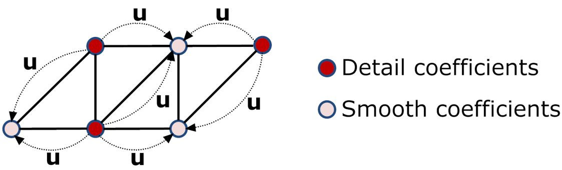

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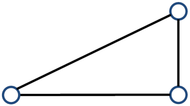

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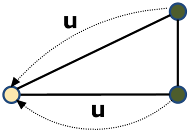

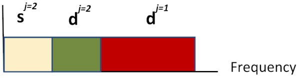

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- Let us try to answer these questions!!

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Graph Construction

Graph-Based Representation of a Signal

- In our case...
  - Samples $\longrightarrow$ Nodes
  - Links $\longrightarrow$ Capture correlation between nodes
- Several graph representations. **Challenge: how to link nodes to accurately capture correlation?**

Graph Weighting

- Weights the links between nodes
- **Affects different processes of the transform!!**

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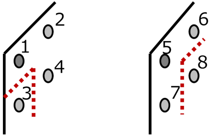
Directional Interpretation

- Graph representation defines filtering directions
- Weighting gives more or less importance to the defined directions

Example: Graph Construction From a Video Signal

- **Example** of the description of a video sequence as a weighted graph of connected pixels
  - Link pixels:



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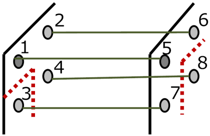

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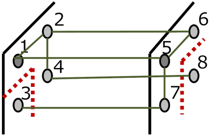

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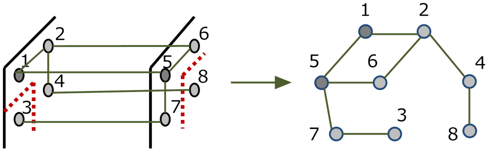

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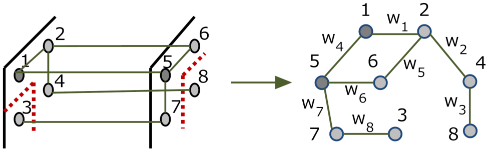

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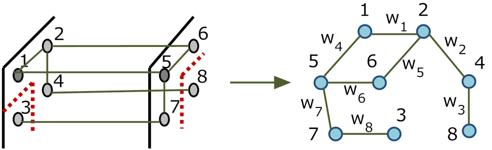

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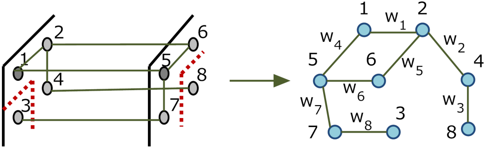

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How to find the weights?

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- How to optimize the weights?

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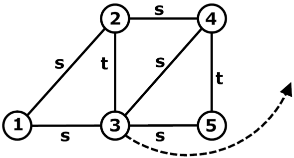

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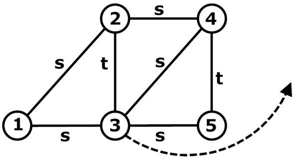

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Definition (Weighted predictor)

- Mean value of the **spatial** neighbors of node *i*:

$$\bar{x}_i^s = \frac{1}{|\mathcal{N}_i|^S} \sum_{j \in \mathcal{N}_i^S} x_j. \tag{1}$$

  - $\mathcal{N}_i^S$ is the set of spatial neighbors of *i*

- Linear predictor of *i*:

$$\hat{x}_i = w_s \bar{x}_i^s + w_t \bar{x}_i^t. \tag{2}$$

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Problem (Optimal weighting problem)

- *Given a graph representation $\mathcal{G} = (\mathcal{V}, \mathcal{E})$: Find the optimal graph weights (w_s and w_t) that minimize the quadratic prediction error (**assuming one-hop prediction filters defined in previous slide**) over all the nodes $i \in \mathcal{V}$:*

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Optimal weights solution

$$\mathbf{w}^* = \begin{bmatrix} w_s^* \\ w_t^* \end{bmatrix} = \begin{bmatrix} \sum_{i \in \mathcal{V}} \bar{x}_i^s \bar{x}_i^s & \sum_{i \in \mathcal{V}} \bar{x}_i^s \bar{x}_i^t \\ \sum_{i \in \mathcal{V}} \bar{x}_i^t \bar{x}_i^s & \sum_{i \in \mathcal{V}} \bar{x}_i^t \bar{x}_i^t \end{bmatrix}^{-1} \cdot \sum_{i \in \mathcal{V}} x_i \begin{bmatrix} \bar{x}_i^s \\ \bar{x}_i^t \end{bmatrix} \tag{3}$$

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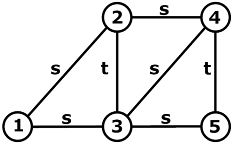

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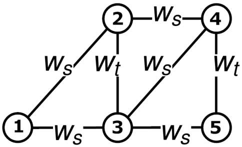

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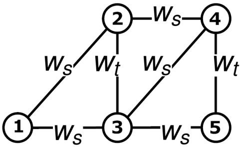

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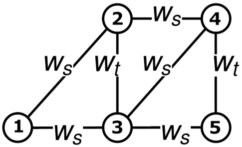

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- We focus on lifting $\implies$ next step: $\mathcal{U}/\mathcal{P}$ assignment

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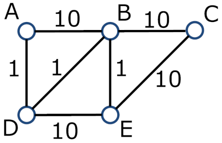

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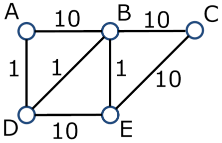

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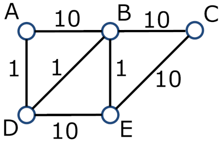

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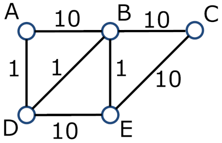

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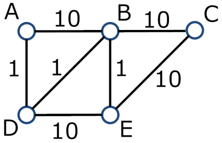

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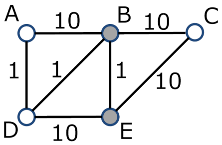

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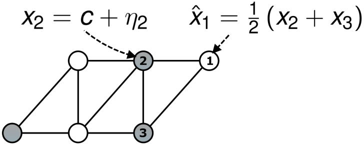

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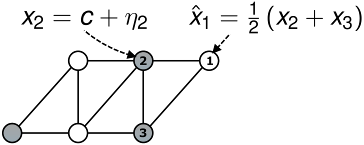

Definition (NM)

Let x_i be a noisy version of some constant c,

$$x_i = c + \eta_i, \tag{5}$$

η_i: independent noise variables with zero mean and variance v

Definition (Unweighted Predictor)

For each node $i \in \mathcal{P}$, consider the predictor

$$\hat{x}_i = \frac{1}{m_i} \sum_{j \in \mathcal{N}_i \cap \mathcal{U}} x_j, \tag{6}$$

- $\mathcal{N}_i$: set of neighbors of node i
- m_i: number of $\mathcal{U}$ neighbors of i ($|\mathcal{N}_i \cap \mathcal{U}|$)

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Proposition (*Noisy Model Prediction Error*)

Let x_i and $\hat{x}_i$ satisfy previous definitions. The total prediction error over all nodes $i \in \mathcal{P}$ is given by

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Corollary (*Optimal Criteria for Fixed $|\mathcal{P}|$ and W*)

For a fixed $|\mathcal{P}|$ and W, the optimal criteria to minimize E_{totNM} over these $\mathcal{P}$ nodes is that **all of them have the same number of $\mathcal{U}$ neighbor nodes**

Let see an example...

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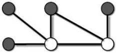

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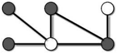

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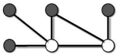

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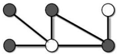

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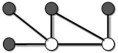

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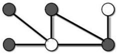

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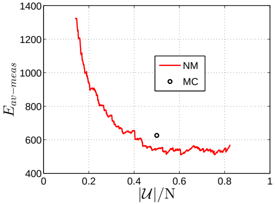

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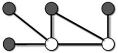

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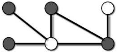

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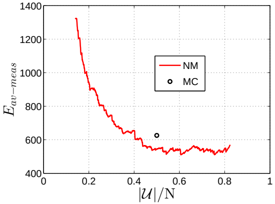

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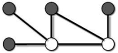

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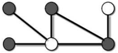

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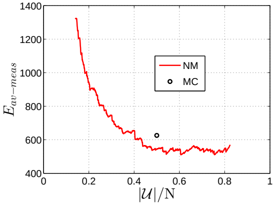

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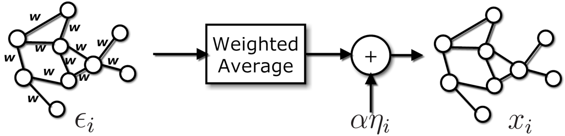

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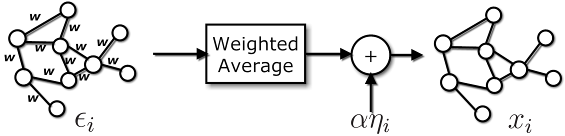

- Predictor: Weighted predictor

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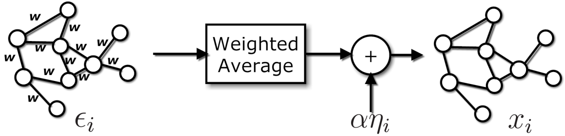

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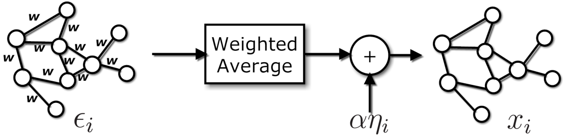

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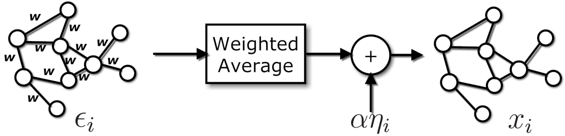

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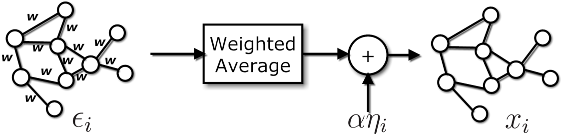

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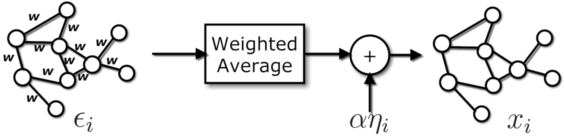

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Proposed Solution Filter design based on the weights of the graph!

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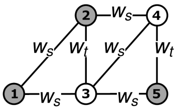
Example Given a weighted graph and an $\mathcal{U}/\mathcal{P}$...



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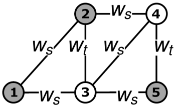

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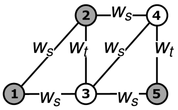

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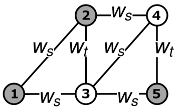

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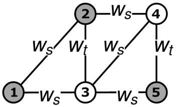

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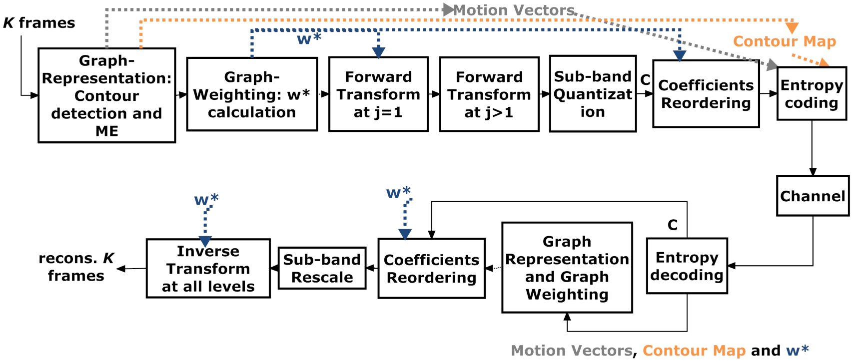
Outline

Encoder and Decoder Data flow

- Apply the transform to video coding leading to a 3-dimensional directional transform
- Proposed coding scheme:



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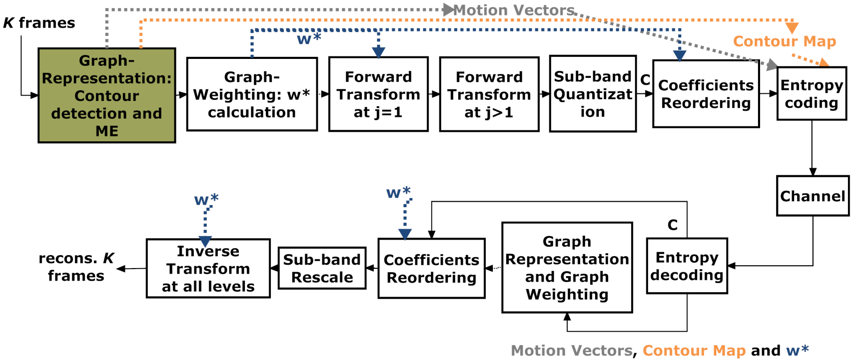

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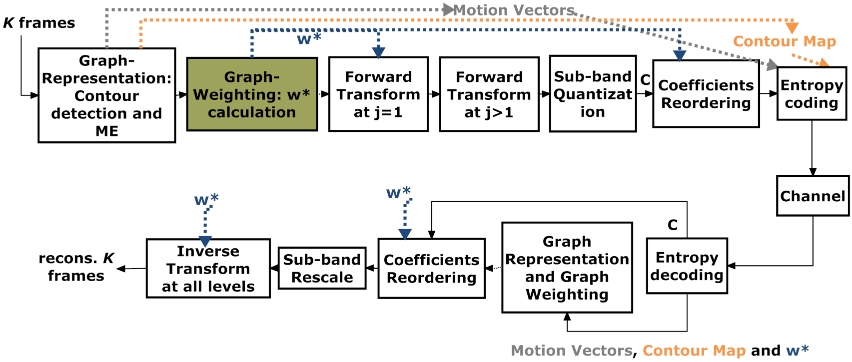

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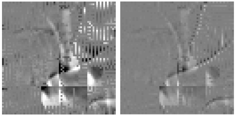

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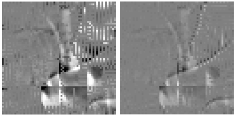

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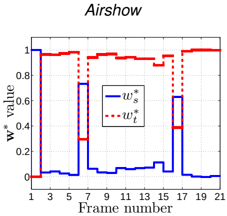

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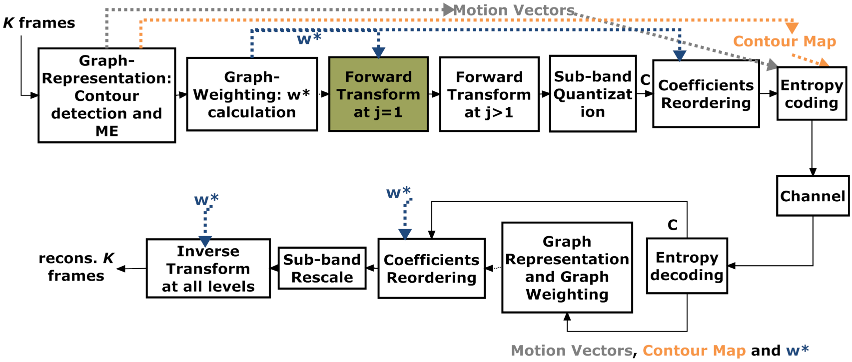

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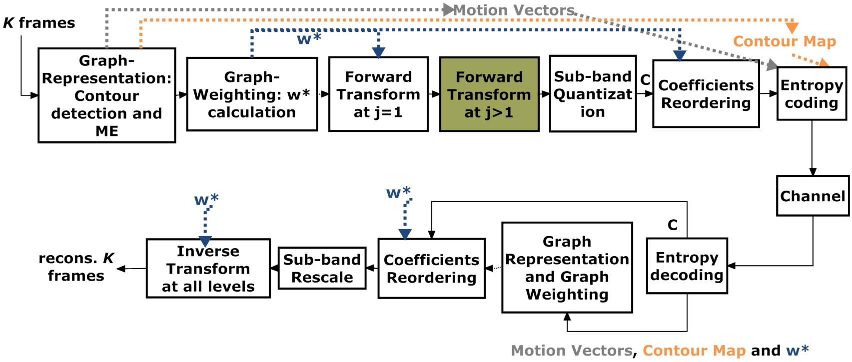

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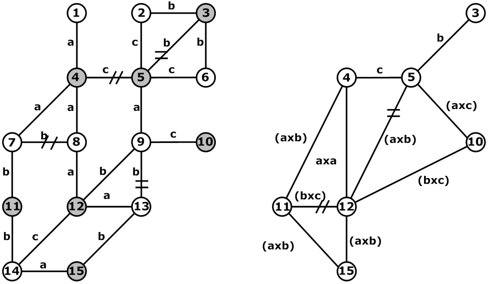

(j-1) level

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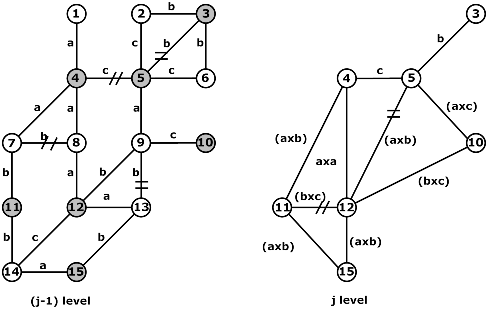

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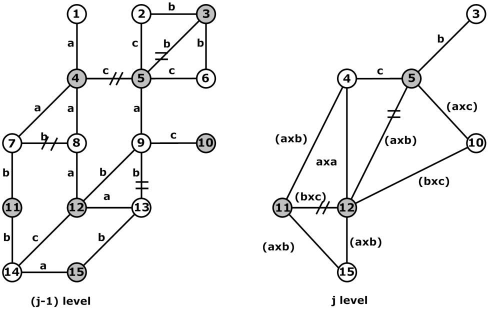

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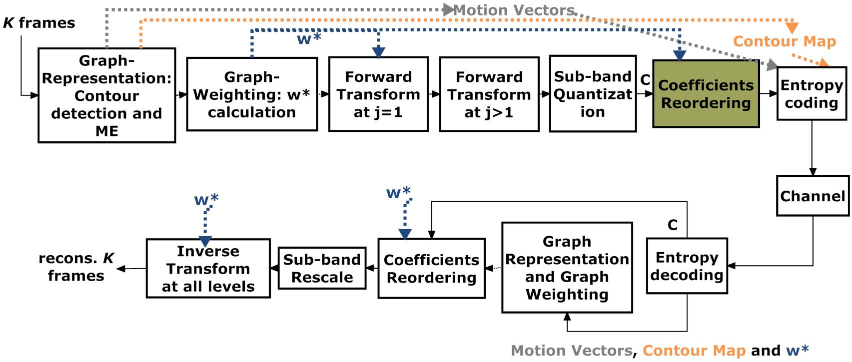

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Proposed Solution Reorder the coefficients using graph information:

1. Inter-subband reordering
2. Intra-subband reordering

Inter-Subband Reordering

Observation Energy in the middle-high frequency subbands will be low

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Proposed reordering Group coefficients that belong to the same subband $\rightarrow$ Long strings of zero coefficients after quantization

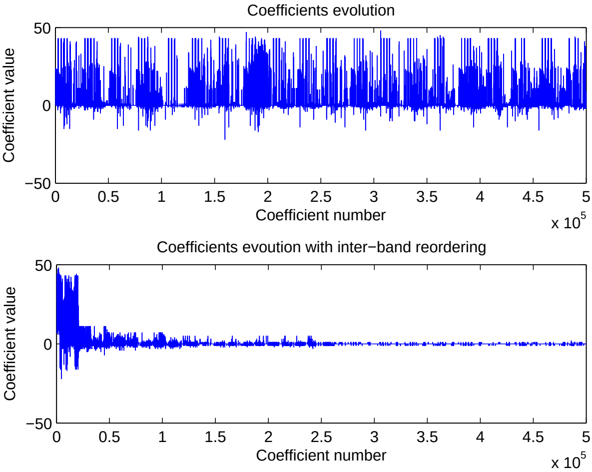
The coefficients will be sorted as:

$$\mathbf{coeffs}_{inter} = \left[\mathbf{s}^{j=N}, \mathbf{d}^{j=N}, \mathbf{d}^{j=N-1}, \ldots, \mathbf{d}^{j=1}\right].$$

$$- \xrightarrow{\textbf{Frequency}} +$$

- $\mathbf{s}^{j=N}$: smooth coefficients at level of decomposition $j = N$ (the lowest frequency subband)
- $\mathbf{d}^{j}$: detail coefficients at generic level of decomposition j

Inter-Subband Reordering Example



20 frames of the sequence *Carphone*

Top: original coefficients. Bottom: reordered coefficients

Intra-Subband Reordering

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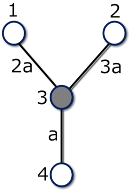
Proposed reordering Reorder the coefficients **in each subband** as a function of the reliability of their links

Intra-Subband Reordering Example



- Detail coefficients (the white nodes) of a generic subband j, $\mathbf{d}^j = [1, 2, 4]$
- Reliability values: $\mathbf{r}^j = [2a, 3a, a]$

Intra-subband reordered coefficients $\mathbf{d}^j_{intra} = [4, 1, 2]$.

$- \xrightarrow{\textbf{Reliability}} +$

- Coefficients energy is lower as the reliability increases

Reordering Results

Test Conditions

- 20 Frames, 2 QCIF sequences
- Comparison: without reordering the coefficients, employing inter-reordering, and inter and intra reordering
- Rate is obtained with an arithmetic coder

Results

	Without reordering	**Inter reordering**	**Inter and Intra reordering**
***Foreman* (32.9 dB)**	503 Kbps	404 Kbps	350 Kbps
***Carphone* (36 dB)**	502 Kbps	425 Kbps	371 Kbps

Experimental Results. Test Conditions

- **Comparison against a motion-compensated DCT video encoder**

	DCT-based	**Proposed**
Coefficients quantization	Uniform dead-zone	Subband dependent
Coefficients scanning	Zigzag scanning order	Inter and Intra reordering
Symbols	Run-length encoding	Run-length encoding
Entropy coding	Arithmetic coder	Arithmetic coder
Block size ME	16×16	16×16

- **Other parameters**:
  - Proposed:
    - Five levels of decomposition
  - DCT-based:
    - 8×8 DCT
- **Side information:**
  - Proposed:
    - MVs differentially encoded with respect to a predicted MV
    - Contour maps $\Longrightarrow$ encoded with JBIG
    - Optimal weights, once every frame. $9 bits/frame$
  - DCT-based:
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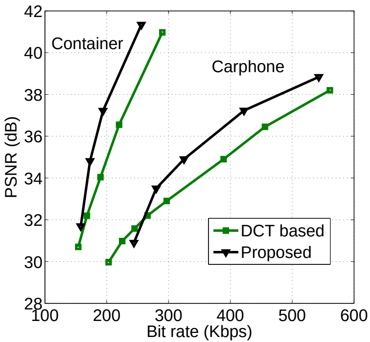

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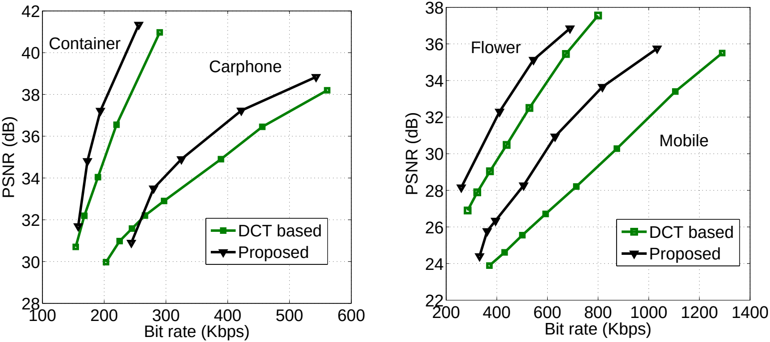

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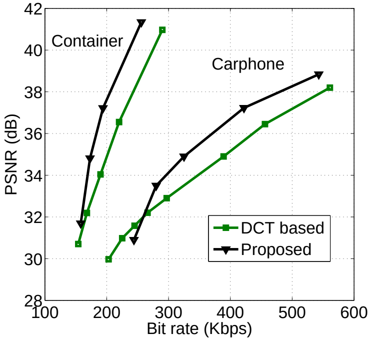

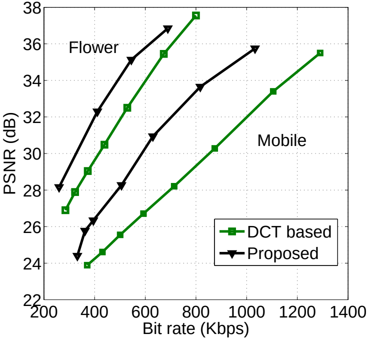

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- 4 dB better in *Mobile*, around 1-2 dB better in *Container*, *Carphone* and *Flower*
- Contour maps side information of about 10*Kbps*; weights side information insignificant

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  - **Results:** Promising results as comparing with the WMC!!! (around 1-2 dBs better in some sequences)

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- Optimization of lifting transforms on graphs in order to obtain a compact representation of the graph signal
- Application to video coding $\Longrightarrow$ Spatial and temporal correlation jointly exploted $\Longrightarrow$ Good results!!

Future Work

- Investigate other models in the $\mathcal{U}/\mathcal{P}$ assignment, as Markov random fields

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- Other applications: multichannel-audio coding, image and video denoising, biomedical signals compact representation

Thank you!

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