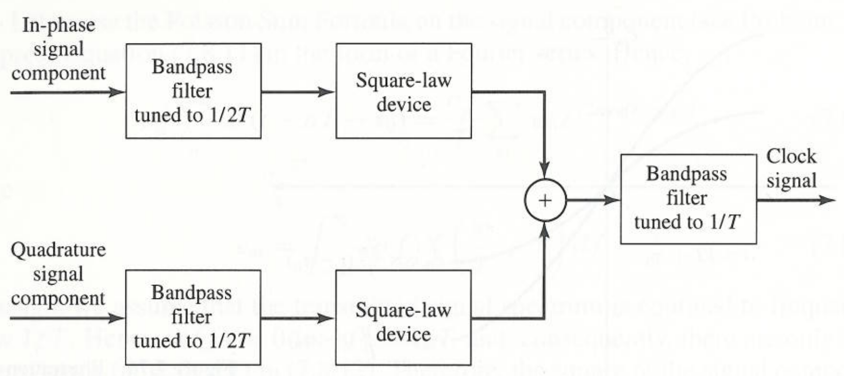

Figure 7.77 Block diagram of timing recovery method for QAM.

two filter outputs are squared (rectified), summed, and then filtered by a narrowband filter tuned to the clock frequency $1/T$. Thus, we generate a sinusoidal signal that is the appropriate clock signal for sampling the outputs of the correlators to recover the information.

In many modern communication systems, the received signal is processed (demodulated) digitally after it has been sampled at the Nyquist rate or faster. In such a case, symbol timing and carrier phase are recovered by signal-processing operations performed on the signal samples. Thus, a PLL for carrier recovery is implemented as a digital PLL and the clock recovery loop of a type described in this section is also implemented as a digital loop. Timing recovery methods based on sampled signals have been described and analyzed by Mueller and Muller (1976).

FURTHER READING

The geometrical representation of digital signals as vectors was first used by Kotelnikov (1947), and by Shannon (1948) in his classic papers. This approach was popularized by Wozencraft and Jacobs (1965). Today this approach to signal analysis and design is widely used. Similar treatments to that given in the text may be found in most books on digital communications.

The matched filter was introduced by North (1943), who showed that it maximized the SNR. Analysis of various binary and M-ary modulation signals in AWGN were performed in the two decades following Shannon's work. Treatments similar to that given in this chapter may be found in most books on digital communications.

A number of books and tutorial papers have been published on the topic of time synchronization. Books that cover both carrier-phase recovery and time synchronization have been written by Stiffler (1971), Lindsey (1972), and Lindsey and Simon (1973), Meyr and Ascheid (1992), and Mengali and D'Andrea (1997). The tutorial paper by Franks (1980) presents a very readable introduction to this topic.

Symbol synchronization for carrier-modulated signals is a topic that has been treated thoroughly and analyzed in many journal articles. Of particular importance are the journal papers by Lyon (1975a,b) that treat timing recovery for QAM signals and the paper by Mueller and Muller (1976) that describes symbol timing methods based on digital processing of signal samples.

PROBLEMS

7.1 Determine the average energy of a set of M PAM signals of the form

$$s_m(t) = s_m\psi(t), \quad m = 1, 2, \ldots, M \qquad 0 \le t \le T$$

where

$$s_m = \sqrt{\mathcal{E}_g}A_m, \quad m = 1, 2, \ldots, M$$

The signals are equally probable with amplitudes that are symmetric about zero and are uniformly spaced with distance d between adjacent amplitudes as shown in Figure 7.11.

7.2 Show that the correlation coefficient of two adjacent signal points corresponding to the vertices of an N-dimensional hypercube with its center at the origin is given by

$$\gamma = \frac{N-2}{N}$$

and their Euclidean distance is

$$d = 2\sqrt{\mathcal{E}_s/N}$$

7.3 Consider the three waveforms $\psi_n(t)$ shown in Figure P-7.3.

1. Show that these waveforms are orthonormal.
2. Express the waveform $x(t)$ as a weighted linear combination of $\psi_n(t)$, $n = 1, 2, 3$, if

$$x(t) = \begin{cases} -1, & 0 \le t \le 1 \\ 1, & 1 \le t \le 3 \\ -1, & 3 \le t \le 4 \end{cases}$$

and determine the weighting coefficients.

7.4 Use the orthonormal waveforms in Problem P-7.3 to approximate the function

$$x(t) = \sin(\pi t/4)$$

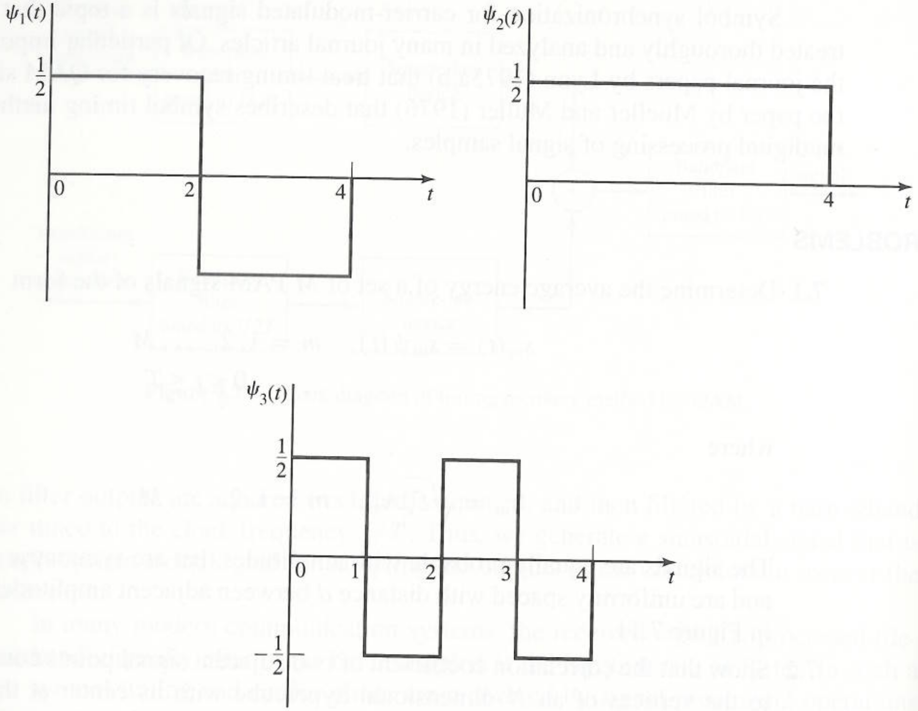

Figure P-7.3

over the interval $0 \leq t \leq 4$ by the linear combination

$$\hat{x}(t) = \sum_{n=1}^{3} c_n \psi_n(t)$$

1. Determine the expansion coefficients $\{c_n\}$ that minimize the mean-square approximation error

$$E = \int_0^4 [x(t) - \hat{x}(t)]^2 \, dt$$

2. Determine the residual mean square error $E_{\min}$.

7.5 Consider the four waveforms shown in Figure P-7.5.

1. Determine the dimensionality of the waveforms and a set of basis functions.
2. Use the basis functions to represent the four waveforms by vectors $\mathbf{s}_1, \mathbf{s}_2, \mathbf{s}_3, \mathbf{s}_4$.
3. Determine the minimum distance between any pair of vectors.

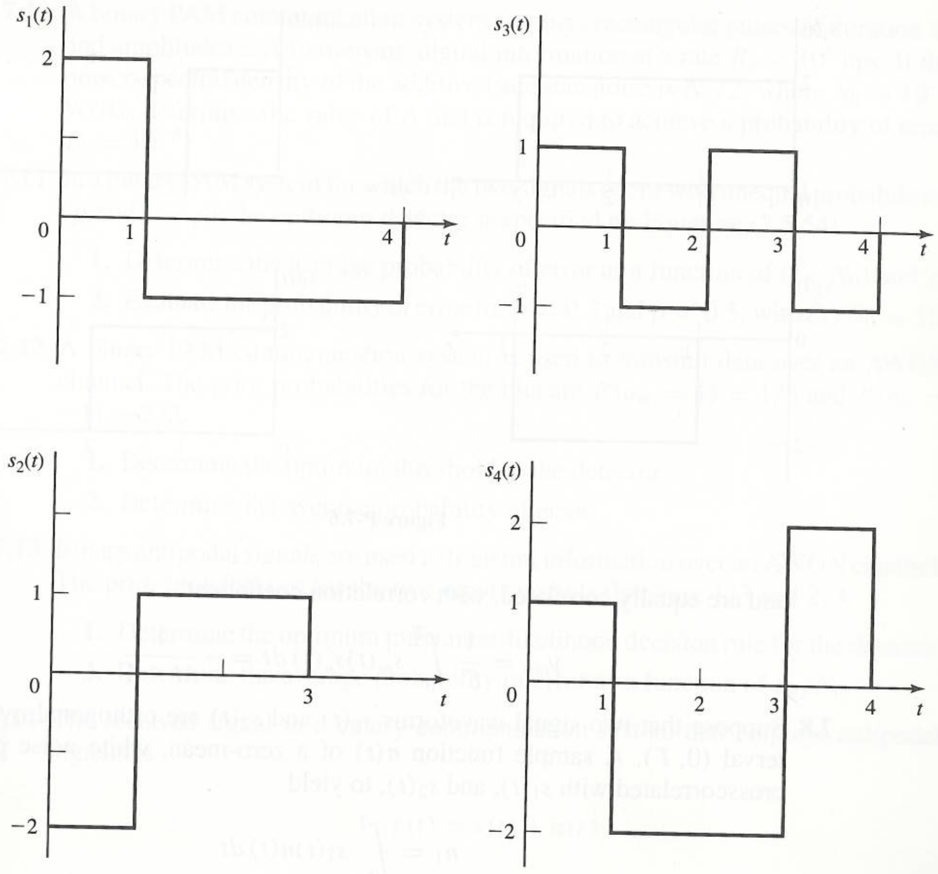

Figure P-7.5

7.6 Determine a set of orthonormal functions for the four signals shown in Figure P-7.6.

7.7 Consider a set of M orthogonal signal waveforms $s_m(t)$, $1 \leq m \leq M$, $0 \leq t \leq T$, all of which have the same energy $\mathcal{E}$. Define a new set of M waveforms as

$$s'_m(t) = s_m(t) - \frac{1}{M} \sum_{k=1}^{M} s_k(t), \quad 1 \leq m \leq M \quad 0 \leq t \leq T$$

Show that the M signal waveform $\{s'_m(t)\}$ have equal energy, given by

$$\mathcal{E}' = (M-1)\mathcal{E}/M$$

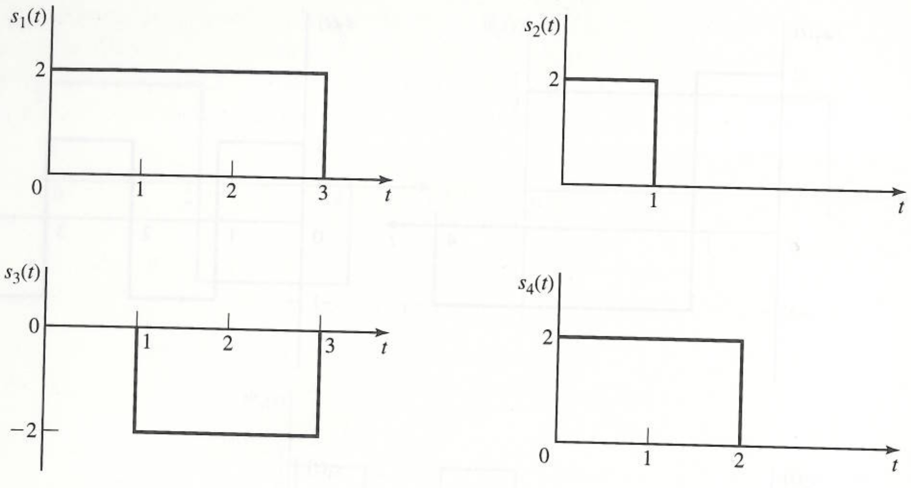

Figure P-7.6

and are equally correlated, with correlation coefficient

$$\gamma_{mn} = \frac{1}{\mathcal{E}'}\int_0^T s'_m(t)s'_n(t)\,dt = -\frac{1}{M-1}$$

7.8 Suppose that two signal waveforms $s_1(t)$ and $s_2(t)$ are orthogonal over the interval $(0, T)$. A sample function $n(t)$ of a zero-mean, white noise process is crosscorrelated with $s_1(t)$, and $s_2(t)$, to yield

$$n_1 = \int_0^T s_1(t)n(t)\,dt$$

$$n_2 = \int_0^T s_2(t)n(t)\,dt$$

Prove that $E(n_1 n_2) = 0$.

7.9 A binary digital communication system employs the signals

$$s_0(t) = 0, \quad 0 \le t \le T$$
$$s_1(t) = A, \quad 0 \le t \le T$$

for transmitting the information. This is called *on-off signaling*. The demodulator crosscorrelates the received signal $r(t)$ with $s_1(t)$ and samples the output of the correlator at $t = T$.

1. Determine the optimum detector for an AWGN channel and the optimum threshold, assuming that the signals are equally probable.
2. Determine the probability of error as a function of the SNR. How does on-off signaling compare with antipodal signaling?

7.10 A binary PAM communication system employs rectangular pulses of duration T_b and amplitudes $\pm A$ to transmit digital information at a rate $R_b = 10^5$ bps. If the power-spectral density of the additive Gaussian noise is $N_0/2$, where $N_0 = 10^{-2}$ W/Hz, determine the value of A that is required to achieve a probability of error $P_b = 10^{-6}$.

7.11 In a binary PAM system for which the two signals occur with unequal probabilities (p and $1-p$), the optimum detector is specified by Equation (7.5.54).

1. Determine the average probability of error as a function of $(\mathcal{E}_b/N_0)$ and p.
2. Evaluate the probability of error for $p = 0.3$ and $p = 0.5$, with $\mathcal{E}_b/N_0 = 10$.

7.12 A binary PAM communication system is used to transmit data over an AWGN channel. The prior probabilities for the bits are $P(a_m = 1) = 1/3$ and $P(a_m = -1) = 2/3$.

1. Determine the optimum threshold at the detector.
2. Determine the average probability of error.

7.13 Binary antipodal signals are used to transmit information over an AWGN channel. The prior probabilities for the two input symbols (bits) are 1/3 and 2/3.

1. Determine the optimum maximum-likelihood decision rule for the detector.
2. Determine the average probability of error as a function of $\mathcal{E}_b/N_0$.

7.14 The received signal in a binary communication system that employs antipodal signals is

$$r(t) = s(t) + n(t)$$

where $s(t)$ is shown in Figure P-7.14 and $n(t)$ is AWGN with power-spectral density $N_0/2$ W/Hz.

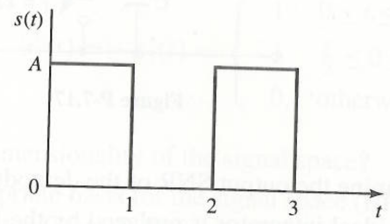

Figure P-7.14

1. Sketch the impulse response of the filter matched to $s(t)$.
2. Sketch the output of the matched filter to the input $s(t)$.

3. Determine the variance the noise of the output of the matched filter at $t = 3$.
4. Determine the probability of error as a function of A and N_0.

7.15 A matched filter has the frequency response

$$H(f) = \frac{1 - e^{-j2\pi fT}}{j2\pi f}$$

1. Determine the impulse response $h(t)$ corresponding to $H(f)$.
2. Determine the signal waveform to which the filter characteristic is matched.

7.16 Prove that when a sinc pulse $g_T(t)$ is passed through its matched filter, the output is the same sinc pulse.

7.17 The demodulation of the binary antipodal signals

$$s_1(t) = -s_2(t) = \begin{cases} \sqrt{\frac{\mathcal{E}_b}{T}}, & 0 \le 1 \le T \\ 0, & \text{otherwise} \end{cases}$$

can be accomplished by use of a single integrator, as shown in Figure P-7.17, which is sampled periodically at $t = kT$, $k = 0, \pm 1, \pm 2, \ldots$. The additive noise is zero-mean Gaussian with power-spectral density of $\frac{N_0}{2}$ W/Hz.

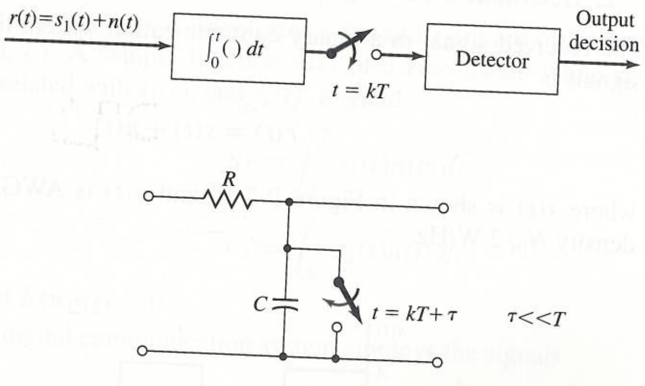

Figure P-7.17

1. Determine the output SNR of the demodulator at $t = T$.
2. If the ideal integrator is replaced by the RC filter shown in Figure P-7.17, determine the output SNR as a function of the time constant RC.
3. Determine the value of RC that maximizes the output SNR.

7.18 Sketch the impulse response of the filter matched to the pulses shown in Figure P-7.18. Also determine and sketch the outputs of each of the matched filters.

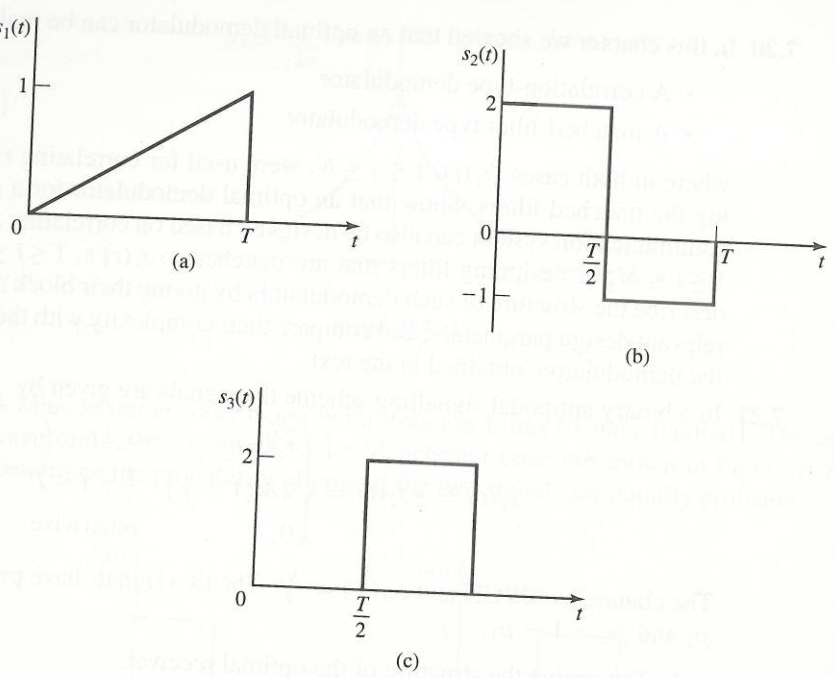

Figure P-7.18

7.19 Three messages m_1, m_2, and m_3 are to be transmitted over an AWGN channel with noise power-spectral density $\frac{N_0}{2}$. The messages are

$$s_1(t) = \begin{cases} 1 & 0 \le 1 \le T \\ 0, & \text{otherwise} \end{cases}$$

$$s_2(t) = -s_3(t) = \begin{cases} 1 & 0 \le t \le \frac{T}{2} \\ -1 & \frac{T}{2} \le 0 \le T \\ 0, & \text{otherwise} \end{cases}$$

1. What is the dimensionality of the signal space?
2. Find an appropriate basis for the signal space (Hint: You can find the basis without using the Gram-Schmidt procedure).
3. Draw the signal constellation for this problem.
4. Derive and sketch the optimal decision regions R_1, R_2, and R_3.
5. Which of the three messages is more vulnerable to errors and why? In other words which of $P(\text{Error} \mid m_i \text{ transmitted})$, $i = 1, 2, 3$ is larger?

7.20 In this chapter we showed that an optimal demodulator can be realized as:

- A correlation-type demodulator
- A matched-filter type demodulator

where in both cases $\psi_j(t)$, $1 \le j \le N$, were used for correlating $r(t)$, or designing the matched filters. Show that an optimal demodulator for a general M-ary communication system can also be designed based on correlating $r(t)$ with $s_i(t)$, $1 \le i \le M$, or designing filters that are matched to $s_i(t)$'s, $1 \le i \le M$. Precisely describe the structure of such demodulators by giving their block diagram and all relevant design parameters, and compare their complexity with the complexity of the demodulators obtained in the text.

7.21 In a binary antipodal signalling scheme the signals are given by

$$s_1(t) = -s_2(t) = \begin{cases} \frac{2At}{T} & 0 \le t \le \frac{T}{2} \\ 2A\left(1 - \frac{t}{T}\right) & \frac{T}{2} \le t \le T \\ 0, & \text{otherwise} \end{cases}$$

The channel is AWGN and $\mathcal{S}_n(f) = \frac{N_0}{2}$. The two signals have prior probabilities p_1 and $p_2 = 1 - p_1$.

1. Determine the structure of the optimal receiver.
2. Determine an expression for the error probability.
3. Plot error probability as a function of p_1 for $0 \le p_1 \le 1$.

7.22 In an additive white Gaussian noise channel with noise power-spectral density of $\frac{N_0}{2}$, two equiprobable messages are transmitted by

$$s_1(t) = \begin{cases} \frac{At}{T}, & 0 \le t \le T \\ 0, & \text{otherwise} \end{cases}$$

$$s_2(t) = \begin{cases} A\left(1 - \frac{t}{T}\right) & 0 \le t \le T \\ 0, & \text{otherwise} \end{cases}$$

1. Determine the structure of the optimal receiver.
2. Determine the probability of error.

7.23 Consider a signal detector with an input

$$r = \pm A + n$$

where $+A$ and $-A$ occur with equal probability and the noise variable n is characterized by the (Laplacian) pdf shown in Figure P-7.23.

1. Determine the probability of error as a function of the parameters A and σ.
2. Determine the "SNR" required to achieve an error probability of 10^{-5}. How does the SNR compare with the result for a Gaussian PDF?

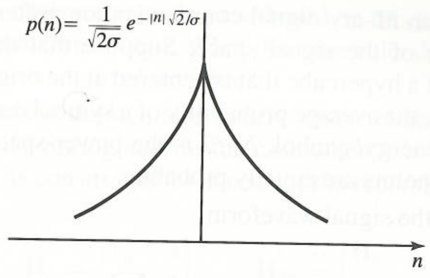

Figure P-7.23

7.24 A Manchester encoder maps an information 1 into 10 and a 0 into 01. The signal waveforms corresponding to the Manchester code are shown in Figure P-7.24. Determine the probability of error if the two signals are equally probable.

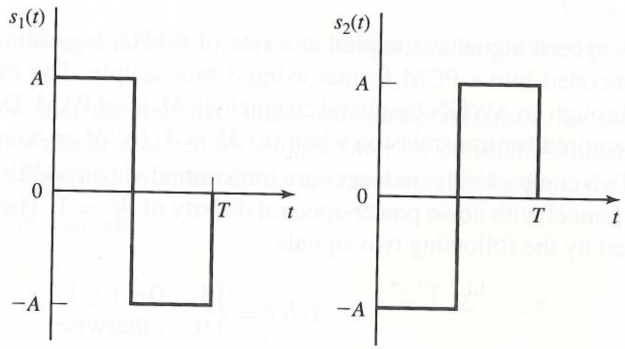

Figure P-7.24

7.25 A three-level PAM system is used to transmit the output of a memoryless ternary source whose rate is 2000 symbols/sec. The signal constellation is shown in Figure P-7.25. Determine the input to the detector, the optimum threshold that minimizes the average probability of error, and the average probability of error.

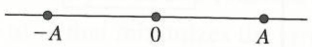

Figure P-7.25

7.26 Consider a biorthogonal signal set with $M = 8$ signal points. Determine a union bound for the probability of a symbol error as a function of $\mathcal{E}_b/N_0$. The signal points are equally likely a priori.

7.27 Consider an M-ary digital communication system where $M = 2^N$, and N is the dimension of the signal space. Suppose that the M signal vectors lie on the vertices of a hypercube that is centered at the origin, as illustrated in Figure 7.29. Determine the average probability of a symbol error as a function of $\mathcal{E}_s/N_0$ where $\mathcal{E}_s$ is the energy/symbol, $N_0/2$ is the power-spectral density of the AWGN, and all signal points are equally probable.

7.28 Consider the signal waveform

$$s(t) = \sum_{k=1}^{n} c_i p(t - nT_c)$$

where $p(t)$ is a rectangular pulse of unit amplitude and duration T_c. The $\{c_i\}$ may be viewed as a code vector $\mathbf{c} = [c_1, c_2, \ldots, c_n]$, where the elements $c_i = \pm 1$. Show that the filter matched to the waveform $s(t)$ may be realized as a cascade of a filter matched to $p(t)$ followed by a discrete-time filter matched to the vector $\mathbf{c}$. Determine the value of the output of the matched filter at the sampling instant $t = nT_c$.

7.29 A speech signal is sampled at a rate of 8 kHz, logarithmically compressed and encoded into a PCM format using 8 bits/sample. The PCM data is transmitted through an AWGN baseband channel via M-level PAM. Determine the bandwidth required for transmission when (a) $M = 4$, (b) $M = 8$, and (c) $M = 16$.

7.30 Two equiprobable messages are transmitted via an additive white Gaussian noise channel with noise power-spectral density of $\frac{N_0}{2} = 1$. The messages are transmitted by the following two signals

$$s_1(t) = \begin{cases} 1 & 0 \le t \le 1 \\ 0, & \text{otherwise} \end{cases}$$

and $s_2(t) = s_1(t-1)$. It is intended to implement the receiver using a correlation type structure, but due to imperfections in the design of the correlators, the structure shown in Figure P-7.30 has been implemented. The imperfection appears in the integrator in the upper branch where instead of $\int_0^1$ we have $\int_0^{1.5}$. The decision

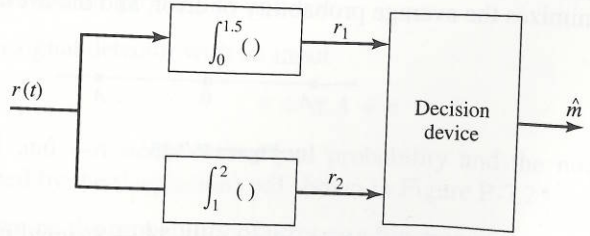

Figure P-7.30

device, therefore, observes r_1 and r_2 and based on this observation has to decide which message was transmitted. What decision rule should be adopted by the decision device for an optimal decision?

7.31 A Hadamard matrix is defined as a matrix whose elements are ± 1 and its row vectors are pairwise orthogonal. In the case when n is a power of 2, an $n \times n$ Hadamard matrix is constructed by means of the recursion

$$\mathbf{H}_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad \mathbf{H}_{2n} = \begin{bmatrix} \mathbf{H}_n & \mathbf{H}_n \\ \mathbf{H}_n & -\mathbf{H}_n \end{bmatrix}$$

1. Let $\mathbf{c}_i$ denote the ith row of an $n \times n$ Hadamard matrix as defined above. Show that the waveforms constructed as

$$s_i(t) = \sum_{k=1}^{n} c_{ik} p(t - kT_c), \quad i = 1, 2, \ldots, n$$

are orthogonal, where $p(t)$ is an arbitrary pulse confined to the time interval $0 \le t \le T_c$.

2. Show that the matched filters (or crosscorrelators) for the n waveforms $\{s_i(t)\}$ can be realized by a single filter (or correlator) matched to the pulse $p(t)$ followed by a set of n crosscorrelators using the code words $\{\mathbf{c}_i\}$.

7.32 The discrete sequence

$$r_k = \sqrt{\mathcal{E}_c} c_k + n_k, \quad k = 1, 2, \ldots, n$$

represents the output sequence of samples from a demodulator, where $c_k = \pm 1$ are elements of one of two possible codewords, $\mathbf{c}_1 = [1, 1, \ldots, 1]$ and $\mathbf{c}_2 = [1, 1, \ldots, 1, -1, \ldots, -1]$. The codeword $\mathbf{c}_2$ has w elements which are $+1$ and $n - w$ elements which are -1, where w is some positive integer. The noise sequence $\{n_k\}$ is white Gaussian with variance σ^2.

1. What is the optimum maximum-likelihood detector for the two possible transmitted signals?
2. Determine the probability error as a function of the parameter $(\sigma^2, \mathcal{E}_b, w)$.
3. What is the value of w that minimizes the error probability?

7.33 A baseband digital communication system employs the signals shown in Figure P-7.33(a) for transmission of two equiprobable messages. It is assumed the communication problem studied here is a "one shot" communication problem, that is, the above messages are transmitted just once and no transmission takes place afterwards. The channel has no attenuation ($\alpha = 1$) and the noise is AWG

1. Find an appropriate orthonormal basis for the representation of the signals.
2. In a block diagram, give the precise specifications of the optimal receiver using matched filters. Label the block diagram carefully.
3. Find the error probability of the optimal receiver.
4. Show that the optimal receiver can be implemented by using just *one* filter [see block diagram shown in Figure P-7.33(b)]. What are the characteristics of the matched filter and the sampler and decision device?
5. Now assume the channel is not ideal, but has an impulse response of $c(t) = \delta(t) + \frac{1}{2}\delta(t - \frac{T}{2})$. Using the same matched filter you used in the previous part, design an optimal receiver.
6. Assuming that the channel impulse response is $c(t) = \delta(t) + a\delta(t - \frac{T}{2})$, where a is a random variable uniformly distributed on [0, 1], and using the same matched filter, design the optimal receiver.

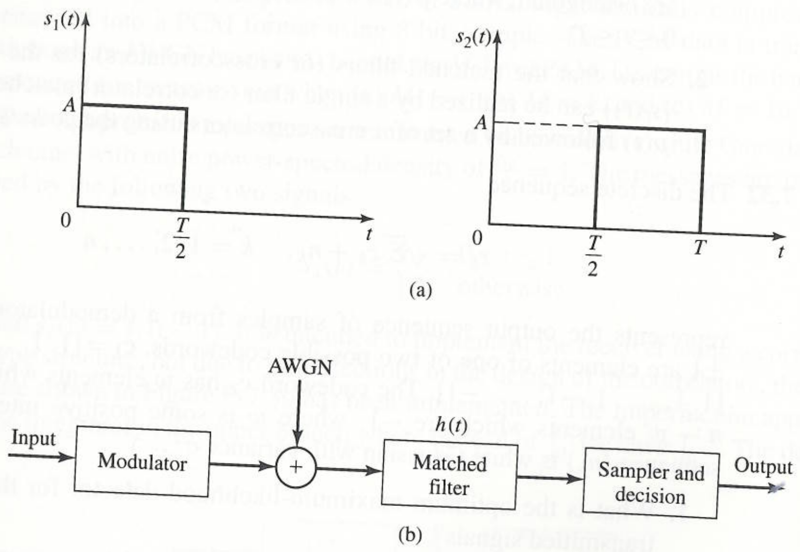

Figure P-7.33

7.34 Suppose that binary PSK is used for transmitting information over an AWGN with power-spectral density of $N_0/2 = 10^{-10}$ W/Hz. The transmitted signal energy is $\mathcal{E}_b = A^2T/2$, where T is the bit interval and A is the signal amplitude. Determine the signal amplitude required to achieve an error probability of 10^{-6}, if the data rate is (a) 10 kbps, (b) 100 kbs, (c) 1 Mbps.

7.35 Consider the signal

$$u(t) = \begin{cases} \frac{A}{T}t\cos 2\pi f_c t, & 0 \le t \le T \\ 0, & \text{otherwise} \end{cases}$$

1. Determine the impulse response of the matched filter for the signal.
2. Determine the output of the matched filter at $t = T$.
3. Suppose the signal $u(t)$ is passed through a correlator which correlates the input $u(t)$ with $u(t)$. Determine the value of the correlator output at $t = T$. Compare your result with that in part (b).

7.36 A carrier component is transmitted on the quadrature carrier in a communication system that transmits information via binary PSK. Hence, the received signal has the form

$$v(t) = \pm\sqrt{2P_s}\cos(2\pi f_c t + \phi) + \sqrt{2P_c}\sin(2\pi f_c t + \phi) + n(t)$$

where ϕ is the carrier phase and $n(t)$ is AWGN. The unmodulated carrier component is used as a pilot signal at the receiver to estimate the carrier phase.

1. Sketch a block diagram of the receiver, including the carrier-phase estimator.
2. Illustrate mathematically the operations involved in the estimation of the carrier-phase ϕ.
3. Express the probability of error for the detection of the binary PSK signal as a function of the total transmitted power $P_T = P_s + P_c$. What is the loss in performance due to the allocation of a portion of the transmitted power to the pilot signal? Evaluate the loss for $P_c/P_T = 0.1$.

7.37 In the demodulation of a binary PSK signal received in white Gaussian noise, a phase-locked loop is used to estimate the carrier-phase ϕ.

1. Determine the effect of a phase error $\phi - \hat{\phi}$ on the probability of error.
2. What is the loss in SNR if the phase error $\phi - \hat{\phi} = 45°$?

7.38 Suppose that the loop filter [see Equation (5.2.4)] for a PLL has the transfer function

$$G(s) = \frac{1}{s + \sqrt{2}}$$

1. Determine the closed-loop transfer function $H(s)$ and indicate if the loop is stable.
2. Determine the damping factor and the natural frequency of the loop.

7.39 Consider the PLL for estimating the carrier phase of a signal in which the loop filter is specified as

$$G(s) = \frac{K}{1 + \tau_1 s}$$

1. Determine the closed-loop transfer function $H(s)$ and its gain at $f = 0$.
2. For what range of value of τ_1 and K is the loop stable?

7.40 The loop filter $G(s)$ in a PLL is implemented by the circuit shown in Figure P-7.40. Determine the system function $G(s)$ and express the time constants τ_1 and τ_2 [see Equation (5.2.4)] in terms of the circuit parameters.

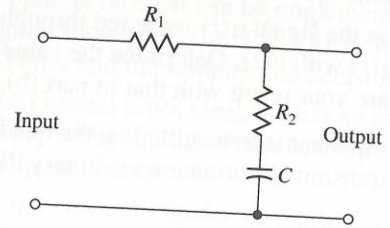

Figure P-7.40

7.41 The loop filter $G(s)$ in a PLL is implemented with the active filter shown in Figure P-7.41. Determine the system function $G(s)$ and express the time constants τ_1 and τ_2 [see Equation (5.2.4)] in terms of the circuit parameters.

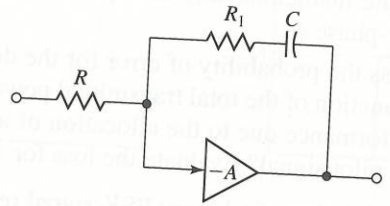

Figure P-7.41

7.42 Consider the four-phase and eight-phase signal constellations shown in Figure P-7.42. Determine the radii r_1 and r_2 of the circles, such that the distance between two adjacent points in the two constellations is d. From this result, determine the additional transmitted energy required in the 8-PSK signal to achieve the same error probability as the four-phase signal at high SNR, where the probability of error is determined by errors in selecting adjacent points.

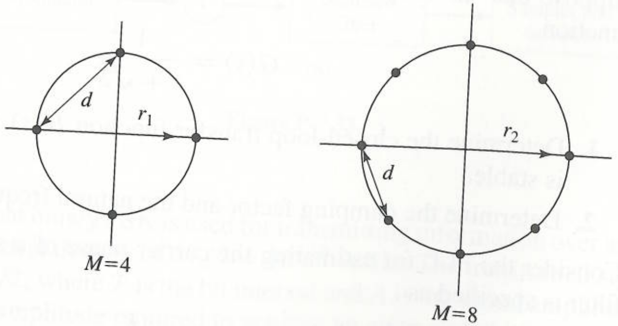

Figure P-7.42

7.43 Consider the two 8-point QAM signal constellation shown in Figure P-7.43. The minimum distance between adjacent points is $2A$. Determine the average transmitted power for each constellation assuming that the signal points are equally probable. Which constellation is more power efficient?

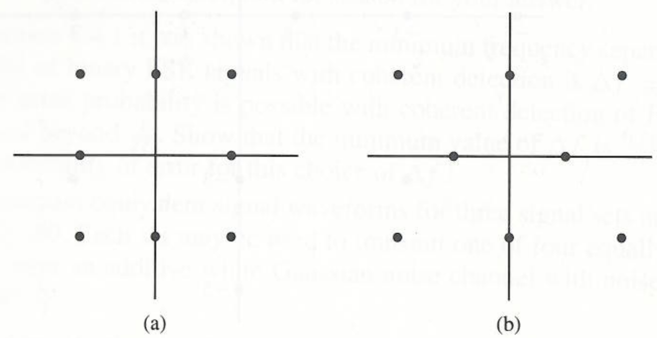

Figure P-7.43

7.44 The 16-QAM signal constellation shown in Figure P-7.44 is an international standard for telephone-line modems (called V.29). Determine the optimum decision boundaries for the detector, assuming that the SNR is sufficiently high so that errors only occur between adjacent points.

7.45 Specify a Gray code for the 16-QAM V.29 signal constellation shown in Problem 7.44.

7.46 Consider the octal signal-point constellations in Figure P-7.46.

1. The nearest neighbor signal points in the 8-QAM signal constellation are separated in distance by A units. Determine the radii a and b of the inner and outer circles.
2. The adjacent signal points in the 8-PSK are separated by a distance of A units. Determine the radius r of the circle.
3. Determine the average transmitter powers for the two signal constellations and compare the two powers. What is the relative power advantage of one constellation over the other? (Assume that all signal points are equally probable).

7.47 Consider a digital communication system that transmits information via QAM over a voice-band telephone channel at a rate 2400 symbols/second. The additive noise is assumed to be white and Gaussian.

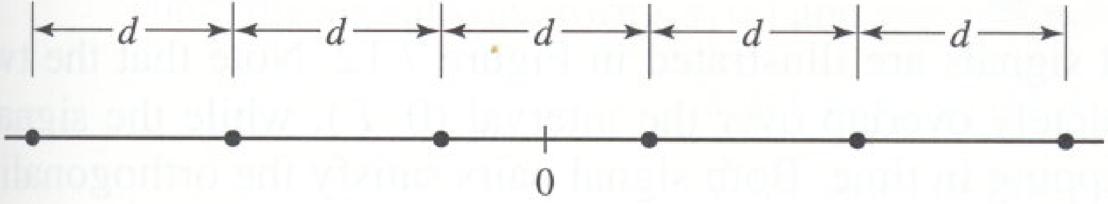

Figure 7.11 Signal points (constellation) for symmetric PAM.