Departamento de Teoría de la Señal y Comunicaciones **23/01/06**
UCIIIM

DIGITAL SIGNAL PROCESSING

THEORY
(Time: 60 minutes. Points: 3/8)

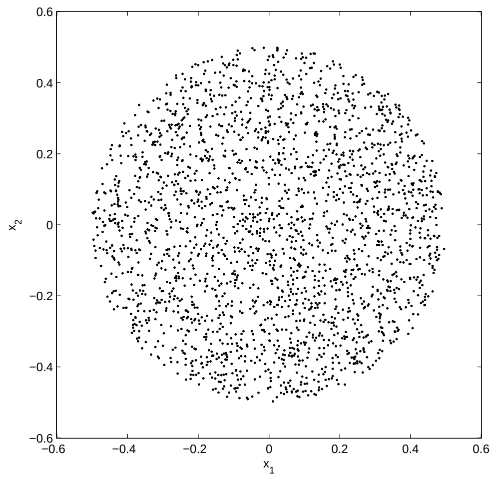
T1.- Consider a decision problem with J hypotheses (exhaustive and excluding) $\{H_j\}_{j=0}^{J-1}$. Suppose that only the costs of making a wrong decision are non-zero, and equal among them, and also that the hypotheses are equiprobable (ML situation). Will the results of the (J) binary tests "one against all" (H_j vs. $\bar{H}_j = \bigcup_{j' \neq j} H_{j'}$) provide the indication for the optimum decision of the ternary problem ?

(20 min; 1p)

T2.- There exist clustering algorithms which consider an a priori number of clusters, and some others which fix this number during the training phase. Describe an example of an algorithm of each type.

(20 min; 1 p)

T3.- Suppose that a unidimensional SO(F)M network (the neurons in the hidden layer have an in-line architecture instead of a bidimensional structure) is trained using as input data the patterns shown in the figure below:



a) Represent the network architecture considering that there are 10 neurons in the hidden layer. Point out carefully the input values and the weights.
b) Draw on the picture a possible development of the SO(F)M. Point out in the figure where the weights of the SO(F)M network are.

(20 min; 1 p)

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PROBLEMS

(Time: 135 minutes. Points: 5/8)

P1.- The following data have been observed:

k	$x^{(k)}$	$s^{(k)}$
1	1	-2
2	2	-1
3	3	1
4	4	2

We wish to estimate s based on x through a linear regression ($\hat{s} = wx + w_0$).

a) Determine the minimum square error regression coefficients.

b) Taking into account the following estimator:

$$\hat{s} = a\cos\left(\frac{\pi}{3}x + \phi_0\right)$$

Determine the values for a and ϕ_0 that minimize the square error given by:

$$E = \sum_{k=1}^{4}\left(y^{(k)} - \hat{y}^{(k)}\right)^2$$

(Hint: it is convenient to apply the equality cos(a+b) = cos(a)cos(b) - sen(a)sen(b) and pose the problem in such a way that linear regression techniques can be used).

c) Point out which estimator fits better the data.

d) Consider now that the frequency of the fitting sinusoid is also unknown. We wish to fit the data to the function:

$$\hat{s} = a\cos(\omega x + \phi_0)$$

Determine the sequential gradient algorithm for the estimation of ω based on the data for minimum MSE. On top of that, comment on the convenience of estimating ω in this case, based on the results obtained for ω = π/3.

(60 min; 2.5 p)

P2.- Consider the observation

$$x = s + r$$

of signal s embedded in noise r. The signal follows the unilateral exponential distribution $E_{\{1,a\}}$

$$p(s) = a \exp(-as)\, u(s)$$

and the noise, which is independent from s, follows the Erlang (Gamma) distribution $E_{\{2,a\}}$

$$p(r) = a^2 r \exp(-ar) u(r)$$

a) Determine $\hat{s}_{ms}$.

b) Calculate the mean $E\{s-\hat{s}\}$ and the variance $Var\{s-\hat{s}\}$ of the error, and comment about the influence of a.

c) Determine $p(\hat{s}_{ms}|s)$.

(75 min; 2.5 p)