Department of Signal Theory and Communications **06/26/08**
UCIIIM

DIGITAL SIGNAL PROCESSING

THEORY

(Time: 60 minutes. Mark: 3/8)

T1.- Consider the linear mean square error (mse) estimation of a random variable s from a random variable x_1. The following statistical information is known:

$$E\{x_1\} = 0, \quad E\{s\} = 1,$$
$$E\{x_1^2\} = 1, \; E\{sx_1\} = 2$$

a) Which of the two following mse designs will incur in a smaller mean square error?

$$\hat{s}_a = w_{0a} + w_{1a}x_1$$
$$\hat{s}_b = w_{1b}x_1$$

b) If we have now access to a second random variable x_2 satisfying:

$$E\{x_2\} = 1, \quad E\{x_2^2\} = 2,$$
$$E\{x_1x_2\} = 1/2, \; E\{sx_2\} = 2,$$

justify if the estimator

$$\hat{s}_c = w_{0c} + w_{1c}x_1 + w_{2c}x_2$$

will or will not provide a smaller mean square error than the estimators proposed in section a).

(20 min; 1p)

T2.- Explain how to carry out the estimation of a pdf by means of a histogram, and how the width of the bars of the histogram influences the behavior of such estimator.

(15 min; 1 p)

T3.- We wish to estimate the variance v of a zero-mean random variable x, using K independent observations of the random variable, $\left\{x^{(k)}\right\}_{k=1}^{K}$. The following estimator is used:

$$\hat{v} = \frac{1}{K}\left[\sum_{k=1}^{K} x^{(k)}\right]^2$$

a) Obtain the bias of this estimator.

b) For $K = 2$, and assuming $E\left\{x^4\right\} = \alpha$, calculate the variance of $\hat{v}$.

(25 min; 1 p)

DIGITAL SIGNAL PROCESSING

PROBLEMS

(Time: 180 minutos. Mark: 5/8)

P1.- Consider the situation described by

$$x = s + n$$

where s and n are two independent random variables with probability density functions:

$$s : G_{\{m, v_s\}}(s)$$
$$n : G_{\{0, v_n\}}(n)$$

a) Obtain the conditional pdf $p(s|x)$.

b) The interval estimator of a random variable s from an observation x can be defined as the sorted pair of values $\{s_a(x), s_b(x)\}$ which minimizes $s_b - s_a$ (i.e., the width of the confidence interval $[s_a, s_b]$), satisfying

$$\Pr\{s_a < s < s_b \,|x\} = 1 - \delta$$

where δ ($\delta \ll 1$) is the level of confidence of the interval estimator; $1 - \delta$ is the confidence coefficient.

Find the interval estimator of s with confidence level δ, expressing your result as a function of the percentile $100(1-\delta/2)$ of the random variable γ: $G_{\{0,1\}}(\gamma)$; i.e., the value $\gamma_{1-\delta/2}$ which verifies

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\gamma_{1-\delta/2}} \exp(-\gamma^2/2) d\gamma = 1 - \delta/2$$

c) What happens if $v_n \gg v_s$?

d) What happens if $v_s \gg v_n$?

(90 min; 2.5 p)

P2.- Consider the two-dimensional binary decision problem with likelihoods:

$$p(x_1, x_2|H_0)=\begin{cases} \alpha & \text{if } 0 \le x_1 \le 3 \text{ and } 0 \le x_2 \le 2 \\ 0 & \text{otherwise} \end{cases}$$

$$p(x_1, x_2|H_1)=\begin{cases} \beta(x_1 + x_2) & \text{if } x_1, x_2 \ge 0 \quad \text{and} \quad x_1 + x_2 \le 3 \\ 0 & \text{otherwise} \end{cases}$$

a) After calculating the value of constants α and β, depict the decision regions corresponding to an LRT decider. Indicate how these regions change with the value of the threshold of the test.

b) Obtain the marginal probability density functions of x_1 and x_2 under both hypotheses (H_0 and H_1). Are x_1 and x_2 independent under any of the hypotheses?

c) For simplicity, one of the two following classifiers, based on the comparison of one of the observations with a threshold, is used:

$$\text{DEC1: } x_1 \underset{D_1}{\overset{D_0}{\gtrless}} \eta_1 \quad ; \quad \text{DEC2: } x_2 \underset{D_1}{\overset{D_0}{\gtrless}} \eta_2$$

Obtain the probabilities P_{FA} and P_D corresponding to classifiers DEC1 and DEC2, expressing them as functions of the thresholds of such deciders: η_1 and η_2, respectively.

d) Depict the operating characteristic (OC) curves (i.e., the curves that represent P_D as a function of P_{FA}) corresponding to classifiers DEC1 and DEC2. Discuss how the operation point of each classifier changes when modifying the value of the corresponding threshold.

e) In the light of the results, can it be concluded that any of the two proposed classifiers, DEC1 or DEC2, achieves a better performance than the other?

(90 min; 2.5 p)