Departamento de Teoría de la Señal y Comunicaciones **14/09/05**
UCIIIM

DIGITAL SIGNAL PROCESSING

THEORY
(Time: 60 minutes. Points: 3/8)

T1.- We wish to estimate a r.v. s based on another r.v. x. The relationship between them is given by:

$$x = s + n_1 + n_2$$

where $s : G(0, v_s)$, $\begin{bmatrix} n_1 \\ n_2 \end{bmatrix} : G\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}\right)$ and n_1, n_2 are independent from the variable to be estimated.

a) Determine the minimum mean square error linear estimator of s given x.

b) Is there a non-linear estimator which has better performance than the one in a) ? Justify your answer.

(20 min; 1p)

T2.- We wish to estimate a r.v. s through a convex combination of linear estimators as in:

$$\hat{s} = \lambda \hat{s}_1 + (1 - \lambda)\hat{s}_2$$

where $\hat{s}_i = \mathbf{w}_i^T \mathbf{x}$, and λ is a mixture parameter (in the range between 0 and 1) related to a real variable a according to the expression:

$$\lambda = \mathrm{sgm}(a) = \frac{1}{1 + \exp(-a)}$$

a) Determine the sequential gradient rule for the sample-by-sample update of $\mathbf{w}_1$ and $\mathbf{w}_2$, if both vectors are modified in order to minimize the cost functions $C_1 = (s - \hat{s}_1)^2$ and $C_2 = |s - \hat{s}_2|$, respectively.

b) Determine the gradient sequential rule for the parameter a, considering that this variable is updated in order to minimize $C = (s - \hat{s})^2$.

(20 min; 1 p)

T3.- Some clustering algorithms are based on the minimization of the square distortion function given by:

$$\sum_{j=1}^{G} \sum_{\mathbf{x}^{(k)} \in G_j} \left\| \mathbf{x}^{(k)} - \mathbf{m}_j \right\|_2^2$$

where G is the number of clusters, G_j is the sample set of the j-th cluster, $\mathbf{m}_j$ is the cluster center and $\mathbf{x}^{(k)}$ is the k-th sample of the data set.

a) Would it be possible to optimize this cost function by applying directly a gradient method?
b) Would it be possible to use this distortion function to determine the number of clusters?

(20 min; 1 p)

DIGITAL SIGNAL PROCESSING

PROBLEMS

(Time: 135 minutes. Points: 5/8)

P1.- Consider the binary decision problem given by the following equiprobable hypotheses:

$$P(x \mid H_0) = (n_0 + 1)(1 - x)^{n_0} \qquad 0 \le x \le 1$$
$$P(x \mid H_1) = (n_1 + 1)x^{n_1} \qquad 0 \le x \le 1$$

where n_0 and n_1 are non-negative integer numbers. Consider the costs $C_{00} = C_{11} = 0$, $C_{01} = 1$, $C_{10} = C > 0$.

a) Observing the shape of the likelihood of the hypotheses, show that the optimum decider (of minimum mean cost) is of the kind:

$$x \underset{D_0}{\overset{D_1}{\gtrless}} \eta$$

for some value of η.

b) Determine the relationship between threshold η and the cost C for the case $n_0 = n_1 = n$.

c) Determine the operating characteristic curve (P_D vs. P_{FA}) as a function of the parameter C, also for the $n_0 = n_1 = n$ case. Show graphically, in an approximate way, cases n=1 and n=2. Comment on the influence of the parameter n.

d) Determine the decider which maximizes the difference P_D-P_{FA}.

(60 min; 2.5 p)

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P2.- Consider a r.v. x with probability distribution given by:

$$p(x \mid R) = \frac{2x}{R} \exp\left(-\frac{x^2}{R} \right) u(x), \quad R > 0$$

K independent observations are made, $\{x^{(k)}\}_{k=1}^{K}$.

a) Determine the maximum likelihood estimator of R, $\hat{R}_{ML}$.

For the estimator obtained in a):

b) Determine its bias.

c) Comment on its consistence.

d) Using the Cramer-Rao bound, comment on its efficiency.

(75 min; 2.5 p)