Grado en Ingeniería de Sist. Audiovisuales, Sist. Com., y Telemática

Communication Theory

(Second year - June 2011)

Last name: ....

Name: ....

ID Number: ....

Group: ....

Has seated the exam

Signature

COMMUNICATION THEORY

Test

(Time: 1 hour. 4/10 Points)

	Grades	
Last name:	C.1	
Name:	C.2	
ID Number: .... Group	C.3	
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Test 1

In a digital communication system we transmit three symbols $\mathcal{A} = \{-1, 0, +1\}$ over an AWGN cannel with power spectral density $N_0/2$. The probability that a 0 is transmitted is twice as high as the one of transmitting either a -1 or a $+1$.

a) Write down the probability of transmitting each symbol.

b) Compute the Maximum Likelihood decision thresholds.

c) Compute the Maximum a Posteriori decision thresholds.

(1.5 points)

Test 2

Let's define the following random process:

$$Y[n] = X[n] + aX[n-1] + bX[n-2]$$

a) Name the sufficient condition over $X[n]$ for $Y[n]$ to be stationary. Detail your answer.

b) Name the sufficient condition over $X[n]$ for $Y[n]$ to be a Gaussian Process. Detail your answer.

c) Compute the autocorrelation of $Y[n]$, assuming that $X[n]$ is a zero-mean stationary process and its autocorrelation function is given by $R_X[n]$.

(1 point)

Test 3

We are transmitting 4 equiprobable symbols (A, B, C y D), and each one of the its encoded with three binary digits that are independently transmitting through a binary erasure channel ($A = 000$, $B = 110$, $C = 101$ y $D = 011$).

a) Compute the probability that we receive 3 zeros, when we transmitted the symbol A.

b) Show that if a single bit is erased by the channel, we can still recover the transmitted symbols.

c) If A was transmitted, compute the probability that we can say at the receiver without error that A was transmitted.

Note: For the binary erasure channel: $p(Y = 0|X = 0) = 1 - \epsilon$, $p(Y =?|X = 0) = \epsilon$, $p(Y =?|X = 1) = \epsilon$ and $p(Y = 1|X = 1) = 1 - \epsilon$

(1.5 points)

COMMUNICATION THEORY

Exercises

(Time: 120 minutes. 6/10 Points)

Last name: ....

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Exercise 1

In a digital communication channel we transmit two equiprobable symbols over an AWGN channel with $N_0/2$ spectral density. The waveforms used to transmit each symbols are the following:

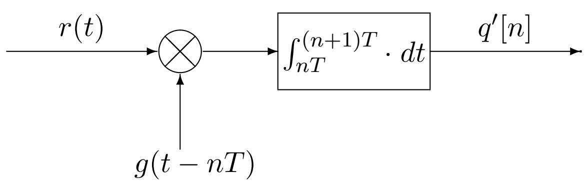
$$s_0(t) = \begin{cases} +1, & \text{si } 0 \le t < \frac{T}{2} \\ -1, & \text{si } \frac{T}{2} \le t < T \\ 0, & \text{otherwise} \end{cases}, \qquad s_1(t) = \begin{cases} +1, & \text{si } 0 \le t < \frac{T}{4} \\ -1, & \text{si } \frac{T}{4} \le t < \frac{T}{2} \\ +1, & \text{si } \frac{T}{2} \le t < \frac{3T}{4} \\ -1, & \text{si } \frac{3T}{4} \le t < T \\ 0, & \text{otherwise} \end{cases}.$$

a) Obtain a base and a constellation that represent the digital system.

b) Compute another set of signals that have the same probability of error, but of minimum mean energy. Compare the energies of both systems.

c) Define an optimal demodulator using correlators. Compute, as well, the optimal decisor and its probability of error.

d) If we use instead the demodulator in the figure with $g(t) = \phi_0(t)$, compute the optimal decider, the probability of error and compare it with the optimal decisor computed in the previous question.



e) Do over the previous question with $g(t) = \phi_0(t) - \phi_1(t)$. Please note that now $g(t)$ might not be unit energy.

(3.0 points)

Exercise 2

We define two communication systems based on two discrete memory-less channels (DMC). System A has as input alphabet $\mathcal{A}_X = \{x_1, x_2\}$ and its output alphabet is denoted by $\mathcal{A}_Y = \{y_1, y_2\}$. The behavior of the whole system is given by the joint probability of the input and output symbols $P_{XY}(x_i, y_j)$ where $i, j \in \{1, 2\}$. System B uses as input alphabet $\mathcal{A}_Y = \{y_1, y_2\}$ and as output alphabet $\mathcal{A}_Z = \{z_1, z_2\}$. The behavior of the whole system is given by the joint probability of the input and output symbols $P_{YZ}(y_i, z_j)$ donde $i, j \in \{1, 2\}$:

$P_{XY}(x_i, y_j)$	x_1	x_2
y_1	α	$(1-\alpha)\epsilon$
y_2	0	$(1-\alpha)(1-\epsilon)$

$P_{YZ}(y_i, z_j)$	y_1	y_2
z_1	$\beta(1-\epsilon')$	0
z_2	$\beta\epsilon'$	$1-\beta$

We want to study the behavior of each system.

a) Compute for System A: $H(X)$, $H(Y)$ and $H(X, Y)$, and for system B: $H(Z)$ and $H(Y|Z)$.

b) For each channel compute, respectively, $P_{Y|X}(y_j|x_i)$ and $P_{Z|Y}(z_j|y_i)$, and sketch the associated DMC for each case. Clearly identify the inputs symbols, the output symbols and the conditional transitions probabilities.

c) Compute the mutual information between the input and output for System A.

We now concatenate the systems A and B, i.e. the output of A is given as input to B.

d) Sketch the joint DMC for both system (from X to Z). Clearly identify the inputs symbols, the output symbols and the conditional transitions probabilities.

d) Fix ϵ and ϵ' so the whole DMC behaves as a binary symmetric channel (BSC).

(3.0 points)