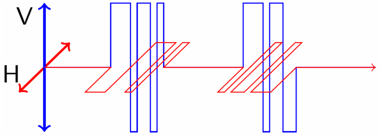
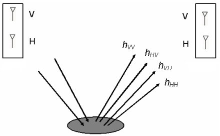
Radar Signal Processing

Instantaneous Radar Polarimetry (Polamouti)
Doppler Resilient Polamouti

Fully Polarimetric Radar

- Fully polarimetric radar systems: Able to transmit and receive in two orthogonal polarizations simultaneously





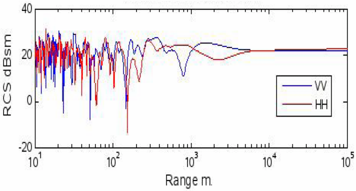
- Scattering matrix:

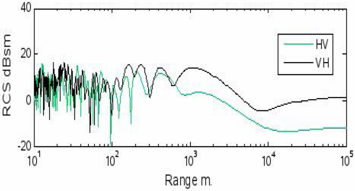
$$\begin{pmatrix} h_{VV} & h_{VH} \\ h_{HV} & h_{HH} \end{pmatrix}$$

h_{VH} is scattering coefficient into vertical polarization channel from horizontally polarized incident field.

Fully Polarimetric Radar: Scattering Matrix

- Raytheon XPatch simulation of polarization scattering matrix for a missile approaching a large complex target





- Is it possible to make polarization scattering matrix available on a pulse by pulse basis at a computational cost comparable to single-channel matched filtering?

Golay Pairs for Radar Polarimetry: Polamouti

Alamouti space-time block code is used to coordinate transmission on V and H channels

- Columns represent different time slots:

$$R = \begin{pmatrix} h_{VV} & h_{VH} \\ h_{HV} & h_{HH} \end{pmatrix} \begin{pmatrix} x & -\widetilde{y} \\ y & \widetilde{x} \end{pmatrix} + Z$$

Idealized model: zero mean Gaussian iid

target covariance matrix $\Lambda = 2\sigma^2 \mathsf{I}_{(2\times 2)}$

noise is zero-mean AWGN with power $2N_0$

- Unitary property: Interplay between Alamouti signal processing and perfect autocoorelation property of Golay pairs

$$\begin{pmatrix} x & -\widetilde{y} \\ y & \widetilde{x} \end{pmatrix} \begin{pmatrix} \widetilde{x} & \widetilde{y} \\ -y & x \end{pmatrix} = \begin{pmatrix} 2L\delta_{k,0} & 0 \\ 0 & 2L\delta_{k,0} \end{pmatrix}$$

Target Detection

Gaussian hypothesis test:

x, y: unit energy pulses

E_t: total transmit energy across two polarization channels

$$\underline{q} = vec\, R \begin{pmatrix} \widetilde{x} & \widetilde{y} \\ -y & x \end{pmatrix} = \begin{cases} 2\sqrt{E_t/4}\,\underline{h} + \underline{n} & : \quad H_1 \\ \underline{n} & : \quad H_0 \end{cases}$$

H_1: hypothesis that target is present

Note: $E[\underline{n}\,\underline{n}^H] = 2N_0 I$

Baseline: Target detection for single channel radar, with total transmit energy E_t

Probability of False Alarm and Probability of Detection

- Energy Detector:

$$(\|\underline{q}\|^2 \sim \chi_8^2) \underset{H_0}{\overset{H_1}{\gtrless}} \gamma$$

- Probability of False Alarm P_F:

$$P_F = Pr(\|\underline{q}\|^2 > \gamma | H_0) = \Phi\left(\frac{\gamma}{2N_0}\right)$$

$$\text{where } \Phi(x) = \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}\right) e^{-x}$$

- Probability of Detection P_D:

$$P_D = Pr(\|\underline{q}\|^2 > \gamma | H_1) = \Phi\left(\frac{\gamma}{2\sigma^2 E_t + 2N_0}\right)$$

Comparison to Single-Channel Radar

Van Trees (Chapter 9): ROC curve for single channel radar

$P_F = P_D^{S+1}$ where $S = \sigma^2 E_t / N_0$ is the SNR at the receiver

$$S' = \text{SNR required by conventional radar to match probability of detection } P_D \text{ for a given probability of false alarm } P_F$$

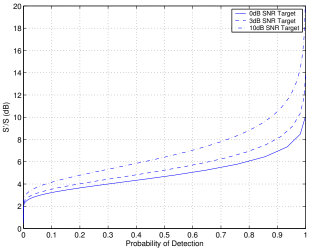
$$= \frac{\log \left[\Phi\left(\frac{\gamma}{2N_0}\right) \Big/ \Phi\left(\frac{\gamma}{2N_0(S+1)}\right) \right]}{\log \Phi\left(\frac{\gamma}{2N_0(S+1)}\right)}$$

where $\Phi(x) = \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}\right) e^{-x}$

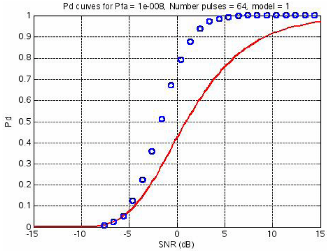
Figure of Merit $= S'/S\ (dB)$

Performance Improvement Is Significant

- Enables radar polarimetry on a pulse by pulse basis.
- Range extension and better target discrimination.



Extra SNR required for a singly-polarized system to get the same probability of detection as the Polamouti system.



Raytheon XPatch simulation shows that gains predicted by the simple model are preserved for a large complex target after pulse integration.

Extension to 4-by-4 case

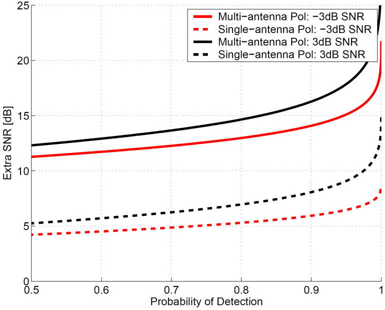
- Alamouti block code is used to coordinate transmission of Golay pairs across polarizations and antennas:

$$W_{4\times 4} = \begin{pmatrix} W_{2\times 2} & -\widetilde{W}_{2\times 2} \\ W_{2\times 2} & \widetilde{W}_{2\times 2} \end{pmatrix}$$

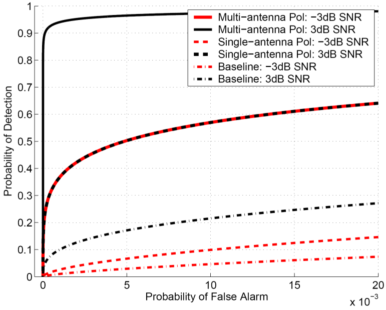
  where $W_{2\times 2}$ is a Polamouti matrix.
- Unitary property:

$$W_{4\times 4}\widetilde{W}_{4\times 4} = (4L\delta_{k,0})I_{4\times 4}$$

Performance Improvement Is Significant



Extra SNR required for the single-channel to get the same PF and PD as Pol. Div. and Pol. & Space Div. systems



Comparison of ROC curves for single-channel, polarization diversity, and multi-antenna polarization diversity systems

Doppler Effect on Polamouti

- Doppler effect over $N = 4$ PRIs:

$$\begin{pmatrix} x_0 & -\widetilde{x}_1 e^{j\theta} & x_2 e^{j2\theta} & -\widetilde{x}_3 e^{j3\theta} \\ x_1 & \widetilde{x}_0 e^{j\theta} & x_3 e^{j2\theta} & \widetilde{x}_2 e^{j3\theta} \end{pmatrix} \begin{pmatrix} \widetilde{x}_0 & \widetilde{x}_1 \\ -x_1 & x_0 \\ \widetilde{x}_2 & \widetilde{x}_3 \\ -x_3 & x_2 \end{pmatrix} \neq \begin{pmatrix} 4L\delta_{k,0} & 0 \\ 0 & 4L\delta_{k,0} \end{pmatrix}$$

- Question: How to zero-force the low-order terms of the Taylor expansions of the diagonal and off-diagonal terms?
- Diagonal term is the same as the composite ambiguity function of Golay pairs (x_0, x_1) and (x_2, x_3).
- PTM sequence? Yes!

Doppler Resilient Polamouti

- Theorem: To zero-force up to $M-1$ Taylor moments of the diagonal and off-diagonal terms, coordinate the transmission of the Alamouti matrices

$$X_0 = \begin{pmatrix} x & -\widetilde{y} \\ y & \widetilde{x} \end{pmatrix} \quad \text{and} \quad X_1 = \begin{pmatrix} -\widetilde{y} & -x \\ \widetilde{x} & -y \end{pmatrix}$$

  according to the length $N = 2^M$ PTM sequence, where 0 locations correspond to X_0 and 1 locations correspond to X_1.

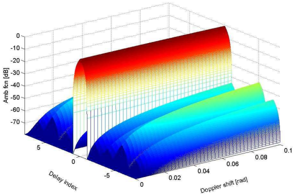
- Example: Zero-forcing three moments

$$X_0 \quad X_1 \quad X_1 \quad X_0 \quad X_1 \quad X_0 \quad X_0 \quad X_1$$

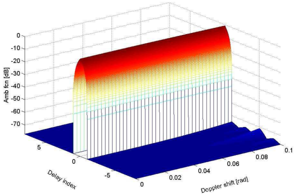
- Over-sampled PTM and Reed-Müller extension are possible.
- Extension to 4 by 4 cases are also possible.

Doppler Resilient Polamouti: Numerical Results

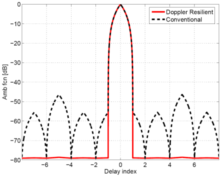
- Zero-forcing 3 moments (diagonal term):



Alternating (conventional)

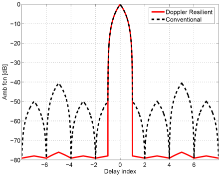


Doppler resilient



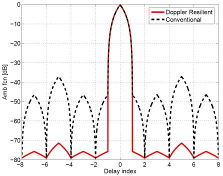
$\theta = 0.025$ rad

24 dB gain



$\theta = 0.05$ rad

28 dB gain

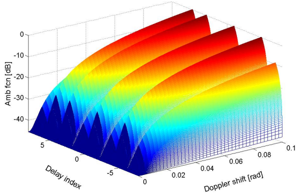


$\theta = 0.075$ rad

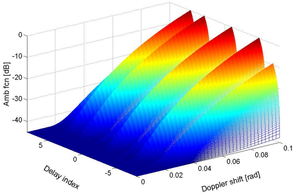
29 dB gain

Doppler Resilient Polamouti: Numerical Results

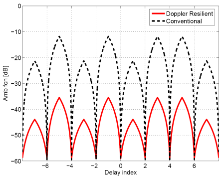
- Zero-forcing 3 moments (off-diagonal term):



Alternating (conventional)

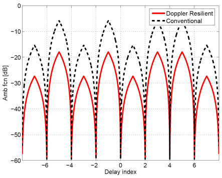


Doppler resilient



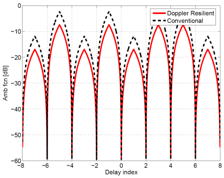
$\theta = 0.025$ rad

24 dB gain



$\theta = 0.05$ rad

12 dB gain



$\theta = 0.075$ rad

5 dB gain

References

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