Radar Signal Processing

Instantaneous Radar Polarimetry (Polamouti)
Doppler Resilient Polamouti
- **Fully polarimetric radar systems**: Able to transmit and receive in two orthogonal polarizations simultaneously.

- **Scattering matrix**: 
  \[
  \begin{pmatrix}
  h_{VV} & h_{VH} \\
  h_{HV} & h_{HH}
  \end{pmatrix}
  \] 
  \(h_{VH}\) is scattering coefficient into vertical polarization channel from horizontally polarized incident field.
Raytheon XPatch simulation of polarization scattering matrix for a missile approaching a large complex target.

Is it possible to make polarization scattering matrix available on a pulse by pulse basis at a computational cost comparable to single-channel matched filtering?
Alamouti space-time block code is used to coordinate transmission on $V$ and $H$ channels

- Columns represent different time slots:

$$R = \begin{pmatrix} h_{VV} & h_{VH} \\ h_{HV} & h_{HH} \end{pmatrix} \begin{pmatrix} x & -\tilde{y} \\ y & \tilde{x} \end{pmatrix} + Z$$

- Idealized model: zero mean Gaussian iid

  target covariance matrix $\Lambda = 2\sigma^2 I_{(2\times2)}$

  noise is zero-mean AWGN with power $2N_0$

- Unitary property: Interplay between Alamouti signal processing and perfect autocorrelation property of Golay pairs

$$\begin{pmatrix} x & -\tilde{y} \\ y & \tilde{x} \end{pmatrix} \begin{pmatrix} \tilde{x} & \tilde{y} \\ -y & x \end{pmatrix} = \begin{pmatrix} 2L\delta_{k,0} & 0 \\ 0 & 2L\delta_{k,0} \end{pmatrix}$$
Gaussian hypothesis test:

\( x, y \): unit energy pulses

\( E_t \): total transmit energy across two polarization channels

\[ q = \text{vec} \ R \begin{pmatrix} \tilde{x} & \tilde{y} \\ -y & x \end{pmatrix} = \begin{cases} 2 \sqrt{E_t/4} h + n & : \ H_1 \\ n & : \ H_0 \end{cases} \]

\( H_1 \): hypothesis that target is present

Note: \( E[n_n^H] = 2N_0 I \)

Baseline: Target detection for single channel radar, with total transmit energy \( E_t \)
Probability of False Alarm and Probability of Detection

- **Energy Detector:**
  \[
  (\|q\|^2 \sim \chi^2)_{\gamma} 
  \]

- **Probability of False Alarm** $P_F$:
  \[
  P_F = Pr(\|q\|^2 > \gamma|H_0) = \Phi \left( \frac{\gamma}{2N_0} \right) 
  \]
  where $\Phi(x) = \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}\right)e^{-x}

- **Probability of Detection** $P_D$:
  \[
  P_D = Pr(\|q\|^2 > \gamma|H_1) = \Phi \left( \frac{\gamma}{2\sigma^2E_t + 2N_0} \right)
  \]
Comparison to Single-Channel Radar

Van Trees (Chapter 9): ROC curve for single channel radar

\[ P_F = P_D^{S+1} \]

where \( S = \sigma^2 E_t / N_0 \) is the SNR at the receiver

SNR required by conventional radar to match probability of detection \( P_D \) for a given probability of false alarm \( P_F \)

\[
\log \left[ \frac{\Phi \left( \frac{\gamma}{2N_0} \right)}{\Phi \left( \frac{\gamma}{2N_0(S+1)} \right)} \right]
\]

\[
\log \Phi \left( \frac{\gamma}{2N_0(S+1)} \right)
\]

where \( \Phi(x) = \left( 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} \right) e^{-x} \)

Figure of Merit \( = S' / S \ (dB) \)
Performance Improvement Is Significant

- Enables radar polarimetry on a pulse by pulse basis.
- Range extension and better target discrimination.

Extra SNR required for a singly-polarized system to get the same probability of detection as the Polamouti system.

Raytheon XPatch simulation shows that gains predicted by the simple model are preserved for a large complex target after pulse integration.
Extension to 4-by-4 case

- Alamouti block code is used to coordinate transmission of Golay pairs across polarizations and antennas:

\[
W_{4 \times 4} = \begin{pmatrix}
W_{2 \times 2} & -\widetilde{W}_{2 \times 2} \\
W_{2 \times 2} & \widetilde{W}_{2 \times 2}
\end{pmatrix}
\]

where \( W_{2 \times 2} \) is a Polamouti matrix.

- Unitary property:

\[
W_{4 \times 4} \widetilde{W}_{4 \times 4} = (4L\delta_{k,0})I_{4 \times 4}
\]
Performance Improvement Is Significant

Extra SNR required for the single-channel to get the same PF and PD as Pol. Div. and Pol. & Space Div. systems

Comparison of ROC curves for single-channel, polarization diversity, and multi-antenna polarization diversity systems
Doppler effect over $N = 4$ PRIs:

$$
\begin{pmatrix}
  x_0 & -\tilde{x}_1 e^{j\theta} & x_2 e^{j2\theta} & -\tilde{x}_3 e^{j3\theta} \\
  x_1 & \tilde{x}_0 e^{j\theta} & x_3 e^{j2\theta} & \tilde{x}_2 e^{j3\theta}
\end{pmatrix}
\begin{pmatrix}
  \tilde{x}_0 & \tilde{x}_1 \\
  -x_1 & x_0 \\
  \tilde{x}_2 & \tilde{x}_3 \\
  -x_3 & x_2
\end{pmatrix}
\neq
\begin{pmatrix}
  4L\delta_{k,0} & 0 \\
  0 & 4L\delta_{k,0}
\end{pmatrix}
$$

**Question:** How to zero-force the low-order terms of the Taylor expansions of the diagonal and off-diagonal terms?

Diagonal term is the same as the composite ambiguity function of Golay pairs $(x_0, x_1)$ and $(x_2, x_3)$.

PTM sequence? Yes!
Theorem: To zero-force up to $M - 1$ Taylor moments of the diagonal and off-diagonal terms, coordinate the transmission of the Alamouti matrices

$$X_0 = \begin{pmatrix} x & -\tilde{y} \\ y & \tilde{x} \end{pmatrix} \quad \text{and} \quad X_1 = \begin{pmatrix} -\tilde{y} & -x \\ \tilde{x} & -y \end{pmatrix}$$

according to the length $N = 2^M$ PTM sequence, where 0 locations correspond to $X_0$ and 1 locations correspond to $X_1$.

Example: Zero-forcing three moments

$$X_0 \ X_1 \ X_1 \ X_0 \ X_1 \ X_0 \ X_0 \ X_1$$

Over-sampled PTM and Reed-Müller extension are possible.

Extension to 4 by 4 cases are also possible.
**Zero-forcing 3 moments (diagonal term):**

- Alternating (conventional)
- Doppler resilient

![Graphs showing Doppler resilient performance compared to conventional for different angles and gains.]

- $\theta = 0.025$ rad
  - 24 dB gain
- $\theta = 0.05$ rad
  - 28 dB gain
- $\theta = 0.075$ rad
  - 29 dB gain
Zero-forcing 3 moments (off-diagonal term):

- \( \theta = 0.025 \text{ rad} \)
  - 24 dB gain
- \( \theta = 0.05 \text{ rad} \)
  - 12 dB gain
- \( \theta = 0.075 \text{ rad} \)
  - 5 dB gain
References


