ELE530: Estimation

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From Detection to Estimation

- We want to know if a coin bias is $P(H) = 0.2$ or $P(H) = 0.3$ or $P(H) = 0.35$.

- We want to know if the bit s is zero or one:

$$x = y \cdot s + w$$

- We want to know if a target is present.

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- We want to know if a coin bias is $P(H) = 0.2$ or $P(H) = 0.3$ or $P(H) = 0.35$.
  - We want to know the coin bias, $P(H)$.
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  - We want to know the gain of the channel y.
- We want to know if a target is present.
  - We want to know the position and velocity of the target.

Minimum Risk

- Detection:

$$R(\delta) = \sum_y \int L(\delta(\mathbf{x}), y) p(\mathbf{x}, y) d\mathbf{x}$$

where $L(\delta(\mathbf{x}), y)$ is a $M \times M$ matrix, measuring the penalty from $\delta(\mathbf{x})$ to y.

- Estimation:

$$R(\delta) = \int L(\delta(\mathbf{x}), y) p(\mathbf{x}, y) d\mathbf{x} dy$$

where $L(\delta(\mathbf{x}), y)$ is function $\mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$, measuring the penalty from $\delta(\mathbf{x})$ to y.

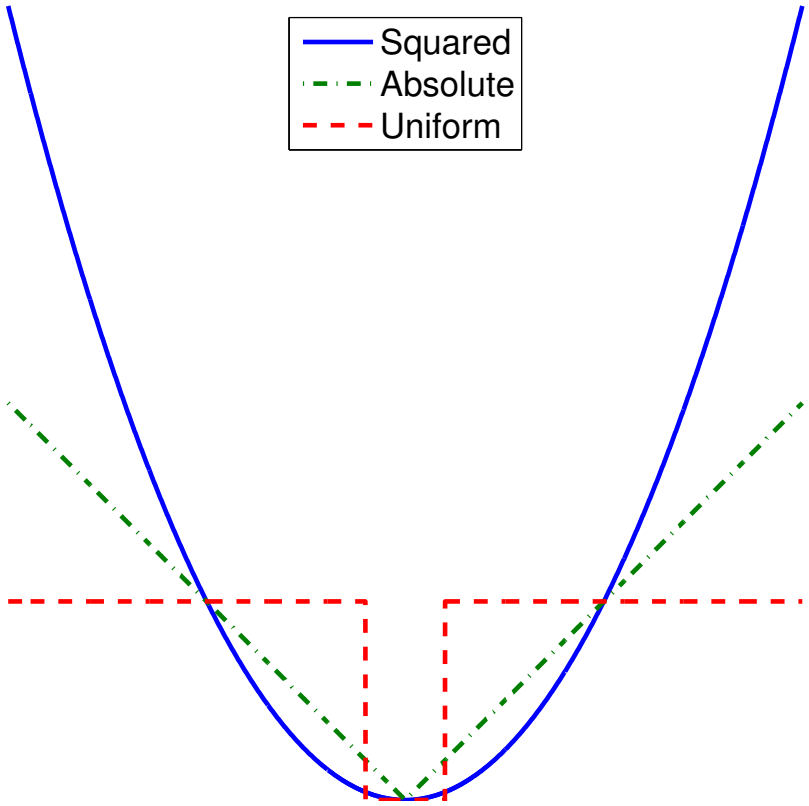
Loss Functions

- Minimum Error:
  - $L(\delta(\mathbf{x}), y) = L(\delta(\mathbf{x}) - y)$.
  - $L(y - \delta(\mathbf{x})) = L(\delta(\mathbf{x}) - y)$.
  - $L(e_1) \leq L(e_2)$ for $|e_1| < |e_2|$.

Loss Functions

- Minimum Error:
  - $L(\delta(\mathbf{x}), y) = L(\delta(\mathbf{x}) - y)$.
  - $L(y - \delta(\mathbf{x})) = L(\delta(\mathbf{x}) - y)$.
  - $L(e_1) \leq L(e_2)$ for $|e_1| < |e_2|$.
- Standard Loss functions:
  - Squared Error: $L(\delta(\mathbf{x}), y) = (\delta(\mathbf{x}) - y)^2$.
  - Absolute Error: $L(\delta(\mathbf{x}), y) = |\delta(\mathbf{x}) - y|$.
  - Uniform Error: $L(\delta(\mathbf{x}), y) = \begin{cases} 0, & |\delta(\mathbf{x}) - y| \leq \Delta/2 \\ 1, & |\delta(\mathbf{x}) - y| > \Delta/2 \end{cases}$.

Loss Functions



Minimum Risk Estimation Rule

- Minimum Risk:

$$\delta^{\text{opt}} = \arg\min_{\delta} \int L(\delta(\mathbf{x}), y) p(\mathbf{x}, y) d\mathbf{x} dy$$
$$= \arg\min_{\delta} \int p(\mathbf{x}) d\mathbf{x} \int L(\delta(\mathbf{x}), y) p(y|\mathbf{x}) dy$$

- We can get the minimum risk rule for each $\mathbf{x}$:

$$\delta^{\text{opt}}(\mathbf{x}) = \arg\min_{\delta(\mathbf{x})} \int L(\delta(\mathbf{x}), y) p(y|\mathbf{x}) dy$$

- This solution is identical to the detection problem. We define a minimum risk estimation rule for each $\mathbf{x}$.

Minimum Mean Squared Error

- Squared error $L(\delta(\mathbf{x}) - y) = (\delta(\mathbf{x}) - y)^2$:

$$\delta^{\text{mmse}}(\mathbf{x}) = \arg\min_{\delta(\mathbf{x})} \int (\delta(\mathbf{x}) - y)^2 p(y|\mathbf{x}) dy$$

Minimum Mean Squared Error

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$$\begin{aligned}\delta^{\text{mmse}}(\mathbf{x}) &= \arg\min_{\delta(\mathbf{x})} \int (\delta(\mathbf{x}) - y)^2 p(y|\mathbf{x}) dy \\ &= \arg\min_{\delta(\mathbf{x})} \delta^2(\mathbf{x}) - 2\delta(\mathbf{x}) \int y p(y|\mathbf{x}) dy + \int y^2 p(y|\mathbf{x}) dy\end{aligned}$$

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\delta^{\text{mmse}}(\mathbf{x}) &= \arg\min_{\delta(\mathbf{x})} \int (\delta(\mathbf{x}) - y)^2 p(y|\mathbf{x}) dy \\
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&= \arg\min_{\delta(\mathbf{x})} \delta^2(\mathbf{x}) - 2\delta(\mathbf{x}) E[y|\mathbf{x}] + \int y^2 p(y|\mathbf{x}) dy
\end{aligned}$$

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- Now we take the derivative with respect to $\delta(\mathbf{x})$:

$$2\delta^{\text{mmse}}(\mathbf{x}) - 2E[y|\mathbf{x}] = 0$$

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- Now we take the derivative with respect to $\delta(\mathbf{x})$:

$$\begin{aligned}2\delta^{\text{mmse}}(\mathbf{x}) - 2E[y|\mathbf{x}] &= 0 \\ \delta^{\text{mmse}}(\mathbf{x}) &= E[y|\mathbf{x}]\end{aligned}$$

- The MMSE solution is the Conditional Mean Estimate.

Minimum Mean Absolute Error

- Absolute error $L(\delta(\mathbf{x}) - y) = |\delta(\mathbf{x}) - y|$:

$$\delta^{\text{mmae}}(\mathbf{x}) = \arg\min_{\delta(\mathbf{x})} \int |\delta(\mathbf{x}) - y| p(y|\mathbf{x}) dy$$

Minimum Mean Absolute Error

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$$\delta^{\text{mmae}}(\mathbf{x}) = \arg\min_{\delta(\mathbf{x})} \int |\delta(\mathbf{x}) - y| p(y|\mathbf{x}) dy$$
$$= \arg\min_{\delta(\mathbf{x})} \int_{-\infty}^{\delta(\mathbf{x})} (\delta(\mathbf{x}) - y) p(y|\mathbf{x}) dy - \int_{\delta(\mathbf{x})}^{\infty} (\delta(\mathbf{x}) - y) p(y|\mathbf{x}) dy$$

Minimum Mean Absolute Error

- Absolute error $L(\delta(\mathbf{x}) - y) = |\delta(\mathbf{x}) - y|$:

$$\begin{aligned}
\delta^{\text{mmae}}(\mathbf{x}) &= \arg\min_{\delta(\mathbf{x})} \int |\delta(\mathbf{x}) - y| p(y|\mathbf{x}) dy \\
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&= \arg\min_{\delta(\mathbf{x})} \int_{-\infty}^{\delta(\mathbf{x})} P(Y \leq y|\mathbf{x}) dy - \int_{\delta(\mathbf{x})}^{\infty} P(Y \leq y|\mathbf{x}) dy
\end{aligned}$$

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- Now we take the derivative with respect to $\delta(\mathbf{x})$:

$$P(Y \leq \delta^{\text{mmae}}(\mathbf{x})|\mathbf{x}) - P(Y > \delta^{\text{mmae}}(\mathbf{x})|\mathbf{x}) = 0$$

Minimum Mean Absolute Error

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- Now we take the derivative with respect to $\delta(\mathbf{x})$:

$$P(Y \leq \delta^{\text{mmae}}(\mathbf{x})|\mathbf{x}) = 1/2$$

- The MMAE solution is the Conditional Median Estimate.

Minimum Mean Uniform Error

- Uniform error $L(\delta(\mathbf{x}), y) = \begin{cases} 0, & |\delta(\mathbf{x}) - y| \leq \Delta/2 \\ 1, & |\delta(\mathbf{x}) - y| > \Delta/2 \end{cases}$:

$$\delta^{\text{mmue}}(\mathbf{x}) = \arg\min_{\delta(\mathbf{x})} \int L(\delta(\mathbf{x}) - y) p(y|\mathbf{x}) dy$$

$$= \arg\min_{\delta(\mathbf{x})} \int_{-\infty}^{\delta(\mathbf{x})-\Delta/2} p(y|\mathbf{x}) dy + \int_{\delta(\mathbf{x})+\Delta/2}^{\infty} p(y|\mathbf{x}) dy$$

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\delta^{\text{mmue}}(\mathbf{x}) &= \arg\min_{\delta(\mathbf{x})} \int L(\delta(\mathbf{x}) - y) p(y|\mathbf{x}) dy \\
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&= \arg\min_{\delta(\mathbf{x})} 1 - [P(Y \leq \delta(\mathbf{x}) + \Delta/2|\mathbf{x}) - P(Y \leq \delta(\mathbf{x}) - \Delta/2|\mathbf{x})]
\end{aligned}$$

Minimum Mean Uniform Error

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\end{aligned}$$

- We can instead maximize:

$$\delta^{\text{mmue}}(\mathbf{x}) = \arg\max_{\delta(\mathbf{x})} \frac{P(Y \leq \delta(\mathbf{x}) + \Delta/2|\mathbf{x}) - P(Y \leq \delta(\mathbf{x}) - \Delta/2|\mathbf{x})}{\Delta}$$

Minimum Mean Uniform Error

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&= \arg\max_{y} p(y|\mathbf{x})
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Minimum Mean Uniform Error

- Uniform error:

$$\delta^{\text{mmue}}(\mathbf{x}) = \arg\min_{\delta(\mathbf{x})} \int_{-\infty}^{\delta(\mathbf{x})-\Delta/2} p(y|\mathbf{x})dy + \int_{\delta(\mathbf{x})+\Delta/2}^{\infty} p(y|\mathbf{x})dy$$

- We can instead maximize:

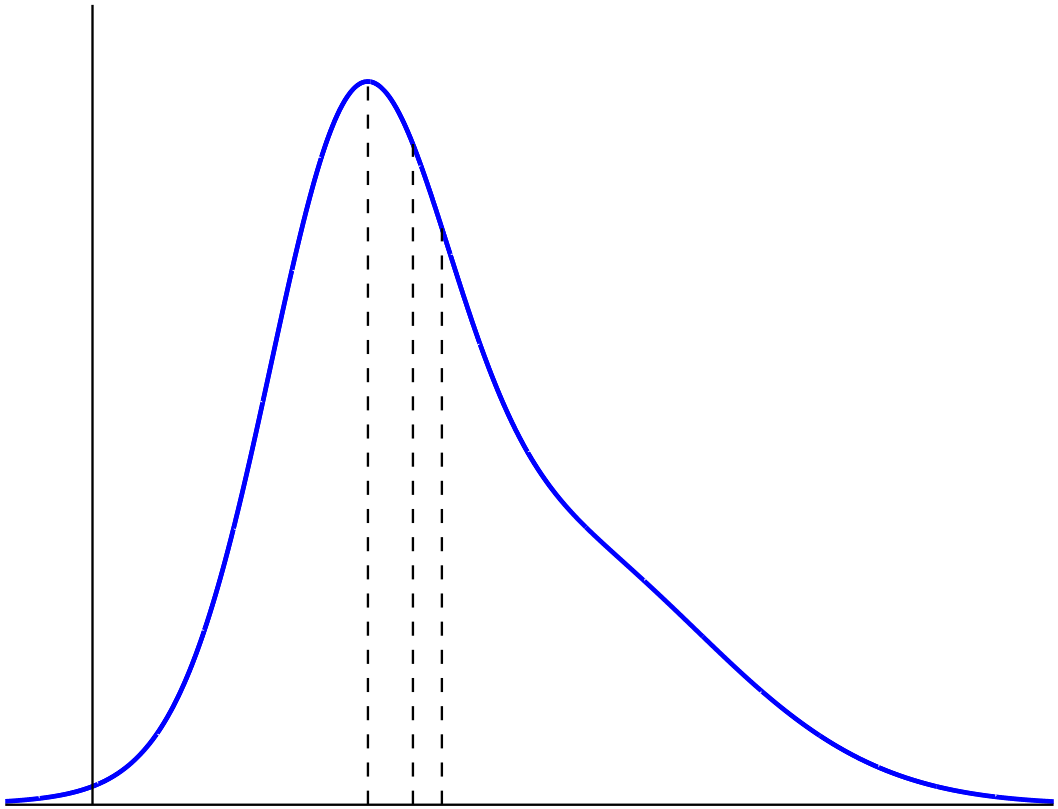
$$\delta^{\text{mmue}}(\mathbf{x}) = \arg\max_{y} p(y|\mathbf{x})$$

- Now we take the derivative with respect to $\delta(\mathbf{x})$:

$$\frac{dp(y|\mathbf{x})}{dy}\Big|_{y=\delta^{\text{mmue}}(\mathbf{x})} = 0$$

- The MMUE solution is the Conditional Mode Estimate.
- The MMUE solution is the Maximum a posteriori (MAP).

Loss Functions



Example

- Estimate the MMSE, MMAE and MMUE rules:

$$p(y) = \alpha e^{-\alpha y} \qquad y \geq 0$$
$$p(x|y) = y e^{-yx} \qquad x \geq 0$$

- First, we compute the joint density:

$$p(x, y) = \alpha y e^{-(\alpha + x)y} \qquad x, y \geq 0$$

- Second, we derive the conditional density for y:

$$p(y|x) = \frac{p(y, x)}{p(x)} = (\alpha + x)^2 y e^{-(\alpha + x)y} \qquad y \geq 0$$

Example: Squared Error

- Estimate the minimum mean squared error rule:

$$
\begin{aligned}
\delta^{\text{mmse}}(x) &= \int_0^\infty y^2(\alpha+x)^2 e^{-(\alpha+x)y} dy \\
&= -y^2(\alpha+x)e^{-(\alpha+x)y}\Big|_0^\infty + 2\int_0^\infty y(\alpha+x)e^{-(\alpha+x)y} dy \\
&= -2ye^{-(\alpha+x)y}\Big|_0^\infty + 2\int_0^\infty e^{-(\alpha+x)y} dy \\
&= -\frac{2}{\alpha+x}e^{-(\alpha+x)y}\Big|_0^\infty \\
&= \frac{2}{\alpha+x}
\end{aligned}
$$

Example: Absolute Error

- Estimate the minimum mean absolute error rule:

$$\int_{\delta^{\text{mmae}}(x)}^{\infty} y(\alpha+x)^2 e^{-(\alpha+x)y} dy = 1/2$$

$$-y(\alpha+x)e^{-(\alpha+x)y}\Big|_{\delta^{\text{mmae}}(x)}^{\infty} + \int_{\delta^{\text{mmae}}(x)}^{\infty} (\alpha+x)e^{-(\alpha+x)y} dy = 1/2$$

$$\delta^{\text{mmae}}(x)(\alpha+x)e^{-(\alpha+x)\delta^{\text{mmae}}(x)} + e^{-(\alpha+x)\delta^{\text{mmae}}(x)} = 1/2$$

$$\delta^{\text{mmae}}(x) = \frac{K}{\alpha+x} \qquad \text{where} \qquad (K+1)e^{-K} = 1/2$$

$$\delta^{\text{mmae}}(x) = \frac{1.6783}{\alpha+x}$$

Example: Uniform Error

- Estimate the minimum mean uniform error rule:

$$\begin{aligned}\frac{dp(y|x)}{dy}\Big|_{y=\delta^{\text{mmue}}(\mathbf{x})} =&(\alpha+x)^2 e^{-(\alpha+x)y}(1-y(\alpha+x))=0\\ &1-\delta^{\text{mmae}}(x)(\alpha+x)=0\\ &\delta^{\text{mmae}}(x)=\frac{1}{\alpha+x}\end{aligned}$$

- Estimators:

MMSE	MMAE	MMUE
$\frac{2}{\alpha+x}$	$\frac{1.6783}{\alpha+x}$	$\frac{1}{\alpha+x}$

Example 2

- Estimate the MMSE, MMAE and MMUE rules:

$$p(y) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} y^{\alpha-1}(1-y)^{\beta-1} \qquad 0 \leq y \leq 1$$
$$p(x|y) = ye^{-yx} \qquad x \geq 0$$

- First, we compute the joint density:

$$p(x,y) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} y^{\alpha}(1-y)^{\beta-1} e^{-xy} \qquad x \geq 0,\ 0 \leq y \leq 1$$

- To derive the conditional density for y, we need

$$p(x) = \int_0^1 \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} y^{\alpha}(1-y)^{\beta-1} e^{-xy} dy$$

Example 2: Uniform Error

- Estimate the minimum mean uniform error rule:

$$\frac{dp(y|x)}{dy}\Big|_{y=\delta^{\text{mmue}}(\mathbf{x})} = \frac{d\frac{p(y,x)}{p(x)}}{dy}\Big|_{y=\delta^{\text{mmue}}(\mathbf{x})} = \frac{dp(y,x)}{dy}\Big|_{y=\delta^{\text{mmue}}(\mathbf{x})}$$
$$= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} y^{\alpha-1}(1-y)^{\beta-2}e^{-xy}(\alpha(1-y)-(\beta-1)y-xy(1-y)) = 0$$

$$\alpha(1-y)-(\beta-1)y-xy(1-y) = 0$$
$$xy^2-(x+\alpha+\beta-1)y+\alpha = 0$$

$$\delta^{\text{mmue}}(\mathbf{x}) = \frac{(x+\alpha+\beta-1)-\sqrt{(x+\alpha+\beta-1)^2-4\alpha x}}{2x}$$

Properties

- Property 1:

  If $L(\delta(\mathbf{x})-y)$ is symmetric and convex, and $p(y|\mathbf{x})$ is symmetric about its conditional mean, then

$$\delta(\mathbf{x}) = \arg\min_{\delta(\mathbf{x})} \int L(\delta(\mathbf{x})-y)p(y|\mathbf{x})dy = \delta^{\text{mmse}}(\mathbf{x})$$

- Property 2:

  If $L(\delta(\mathbf{x})-y)$ is symmetric and nondecreasing, $p(y|\mathbf{x})$ is symmetric about its conditional mean and unimodal, and $\lim_{a\to\infty} L(a)p(a|\mathbf{x}) = 0$, then

$$\delta(\mathbf{x}) = \arg\min_{\delta(\mathbf{x})} \int L(\delta(\mathbf{x})-y)p(y|\mathbf{x})dy = \delta^{\text{mmse}}(\mathbf{x})$$

Nonrandom Parameter Estimation

- Now assume y is an unknown parameter.
- Therefore, there is not a $p(y)$ that characterizes y.
- What can we do?

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  - Is there an equivalent Minimax rule for estimation?

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    No

  - Is there an equivalent Minimax rule for estimation?

    Somewhat.

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    No, because it does it not make sense to impose $P_{fa} \leq \alpha$, there is no P_{fa} or P_M for estimation problems.
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Nonrandom Parameter Estimation

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  - Is there an equivalent Minimax rule for estimation?

    Somewhat. For parameters with finite support we can define $p(y) = d(y - y_o)$ and look for the estimation rule than minimizes the maximum error. Does it makes sense?

Difference Between Classic and Bayesian Statistics