ELE530 Assignment 2

Due Date: March 4th

You can either email your answer to `fp@princeton.edu` before the lecture or hand it in during the lecture. If you use any computer program please email it to me.

1. We receive two possible signals. For Hypothesis H_0:

$$p(x_i|y=0) = x_i^{k-1}\frac{\exp(-x_i/\theta_0)}{\theta_0^k\Gamma(k)}.$$

   For Hypothesis H_1:

$$p(x_i|y=1) = x_i^{k-1}\frac{\exp(-x_i/\theta_1)}{\theta_1^k\Gamma(k)}$$

   (a) Find the rejection region for n samples and $\theta_0 < \theta_1$, for minimum risk and equal prior probability. Find c as a function of k, θ_1 and θ_0 such that $\mathcal{X}_1 = \left\{x\middle|\frac{1}{n}\sum_i x_i > c\right\}$.

   (b) For $k=1$, $\theta_1 = 2$ and $\theta_0 = 1$ and $n = 10$ compute P_{fa} and P_D for the previous threshold. Assume that $\frac{1}{n}\sum_i x_i$ is Gaussianly distributed.

   (c) Generate 10×10^5 samples from $p(x_i|y=0)$ for $k=1$ and $\theta_0 = 1$ and estimate the false alarm rate as the fraction of samples that cross the threshold in a). Compare with the result in the previous section. [Note: You can get samples from a Gamma distribution in Matlab from `gamrnd`.]

(15 Points)

2. We know that y is distributed as:

$$p(x|y) = \frac{x^{k-1}y^k}{\Gamma(k)}\exp(-xy)$$

   We want to know if $y=1$ or $y>1$ and we get a sample x.

(a) Find the rejection region for the locally most powerful test as function of τ and k.

(b) Find the rejection region for the locally most powerful test for $k = 2$ for $\tau = 1$.

(10 Points)

3. We want to detect between 2 signals embedded in Laplacian noise:

$$x_j = s_{ij} + w_j$$

where $s_i \in \pm 1$ and

$$p(w_j) = \frac{1}{2}\exp(-|w_j|)$$

(a) Find A and B for a sequential test with $P_{fa} = 0.01$ and $P_M = 0.01$.

(b) Which is the minimum number of samples we need before we can make a decision?

(c) If $A = \exp(-6)$ and $B = \exp(6)$, what is the probability that we finish when we take three samples? And what is the probability that we are correct? [Hint: It is easier if you work with the log-likelihoods for b) and c)]

(15 Points)