OFDM : N samples per block

$$s^{(n)}[m] = s[nN+m],\ m \in \{0,1,\cdots,N-1\}$$

$$s^{(n)}[m] = \frac{N}{\sqrt{T}} \text{IDFT}_N \{A_0[n], A_1[n], \cdots, A_{N-1}[n]\} = \frac{1}{\sqrt{T}} \sum_{k=0}^{N-1} A_k[n]\ e^{j\frac{2\pi k}{N}m}$$

$$s^{(n)}[m] = \frac{1}{\sqrt{T}} \left(A_0[n]\ \underbrace{e^{j\frac{2\pi\ 0}{N}\ m}}_{e^{j0}=1} + A_1[n]\ \underbrace{e^{j\frac{2\pi\ 1}{N}\ m}}_{e^{j\frac{2\pi}{N}m}} + A_2[n]\ \underbrace{e^{j\frac{2\pi\ 2}{N}\ m}}_{e^{j\frac{4\pi}{N}m}} + \cdots + A_{N-1}[n]\ \underbrace{e^{j\frac{2\pi\ (N-1)}{N}\ m}}_{e^{j\frac{2\pi(N-1)}{N}m}} \right)$$

Example: $N=4$

$$s^{(n)}[m] = \frac{1}{\sqrt{T}} \left(A_0[n]\ \underbrace{e^{j\frac{2\pi\ 0}{4}\ m}}_{e^{j0}=1} + A_1[n]\ \underbrace{e^{j\frac{2\pi\ 1}{4}\ m}}_{e^{j\frac{\pi}{2}m}} + A_2[n]\ \underbrace{e^{j\frac{2\pi\ 2}{4}\ m}}_{e^{j\pi m}} + \cdots A_3[n]\ \underbrace{e^{j\frac{2\pi\ 3}{4}\ m}}_{e^{j\frac{3\pi}{2}m}} \right)$$

Example $N=4$: Serial/Parallel + IDFTs

m	0	1	2	3	4	5	6	7	8	9	10	11
$A[m]$	$+1$	-3	$+1$	$+3$	$+1$	-1	$+1$	-1	-1	$+3$	-1	-3

- Serial / Parallel

n	0	1	2
$A_0[n]$	$+1$	$+1$	-1
$A_1[n]$	-3	-1	$+3$
$A_2[n]$	$+1$	$+1$	-1
$A_3[n]$	$+3$	-1	-3

- Samples of each block: $s^{(n)}[m] = s[nN+m]$, $m \in \{0,1,2,3\}$

$$s^{(0)}[m] = \frac{N}{\sqrt{T}} \text{IDFT}_N \{+1,-3,+1,+3\}$$

$$s^{(1)}[m] = \frac{N}{\sqrt{T}} \text{IDFT}_N \{+1,-1,+1,-1\}$$

$$s^{(2)}[m] = \frac{N}{\sqrt{T}} \text{IDFT}_N \{-1,+3,-1,-3\}$$

First block: $s^{(0)}[m] \equiv \{s[0], s[1], s[2], s[3]\}$

$$s^{(0)}[m] = \frac{N}{\sqrt{T}} \text{IDFT}_N \{A_0[0], A_1[0], A_2[0], A_3[0]\} = \frac{N}{\sqrt{T}} \text{IDFT}_N \{+1, -3, +1, +3\}$$

$$s^{(0)}[0] = \frac{1}{\sqrt{T}} \left(\underbrace{A_0[0]}_{+1} \underbrace{e^{j\frac{2\pi\, 0}{4}\, 0}}_{e^{j0}=1} + \underbrace{A_1[0]}_{-3} \underbrace{e^{j\frac{2\pi\, 1}{4}\, 0}}_{e^{j0}=1} + \underbrace{A_2[0]}_{+1} \underbrace{e^{j\frac{2\pi\, 2}{4}\, 0}}_{e^{j0}=1} + \underbrace{A_3[0]}_{+3} \underbrace{e^{j\frac{2\pi\, 3}{4}\, 0}}_{e^{j0}=1} \right) = \frac{2}{\sqrt{T}}$$

$$s^{(0)}[1] = \frac{1}{\sqrt{T}} \left(\underbrace{A_0[0]}_{+1} \underbrace{e^{j\frac{2\pi\, 0}{4}\, 1}}_{e^{j0}=+1} + \underbrace{A_1[0]}_{-3} \underbrace{e^{j\frac{2\pi\, 1}{4}\, 1}}_{e^{j\pi/2}=+j} + \underbrace{A_2[0]}_{+1} \underbrace{e^{j\frac{2\pi\, 2}{4}\, 1}}_{e^{j\pi}=-1} + \underbrace{A_3[0]}_{+3} \underbrace{e^{j\frac{2\pi\, 3}{4}\, 1}}_{e^{j3\pi/2}=-j} \right) = \frac{-j6}{\sqrt{T}}$$

$$s^{(0)}[2] = \frac{1}{\sqrt{T}} \left(\underbrace{A_0[0]}_{+1} \underbrace{e^{j\frac{2\pi\, 0}{4}\, 2}}_{e^{j0}=+1} + \underbrace{A_1[0]}_{-3} \underbrace{e^{j\frac{2\pi\, 1}{4}\, 2}}_{e^{j\pi}=-1} + \underbrace{A_2[0]}_{+1} \underbrace{e^{j\frac{2\pi\, 2}{4}\, 2}}_{e^{j2\pi}=+1} + \underbrace{A_3[0]}_{+3} \underbrace{e^{j\frac{2\pi\, 3}{4}\, 2}}_{e^{j3\pi}=-1} \right) = \frac{2}{\sqrt{T}}$$

$$s^{(0)}[3] = \frac{1}{\sqrt{T}} \left(\underbrace{A_0[0]}_{+1} \underbrace{e^{j\frac{2\pi\, 0}{4}\, 3}}_{e^{j0}=+1} + \underbrace{A_1[0]}_{-3} \underbrace{e^{j\frac{2\pi\, 1}{4}\, 3}}_{e^{j3\pi/2}=-j} + \underbrace{A_2[0]}_{+1} \underbrace{e^{j\frac{2\pi\, 2}{4}\, 3}}_{e^{j3\pi}=-1} + \underbrace{A_3[0]}_{+3} \underbrace{e^{j\frac{2\pi\, 3}{4}\, 3}}_{e^{j9\pi/2}=+j} \right) = \frac{+j6}{\sqrt{T}}$$

Second block: $s^{(1)}[m] \equiv \{s[4], s[5], s[6], s[7]\}$

$$s^{(1)}[m] = \frac{N}{\sqrt{T}} \text{IDFT}_N \{A_0[1], A_1[1], A_2[1], A_3[1]\} = \frac{N}{\sqrt{T}} \text{IDFT}_N \{+1, -1, +1, -1\}$$

$$s^{(1)}[0] = \frac{1}{\sqrt{T}} \left(\underbrace{A_0[1]}_{+1} \underbrace{e^{j\frac{2\pi\, 0}{4}\, 0}}_{e^{j0}=1} + \underbrace{A_1[1]}_{-1} \underbrace{e^{j\frac{2\pi\, 1}{4}\, 0}}_{e^{j0}=1} + \underbrace{A_2[1]}_{+1} \underbrace{e^{j\frac{2\pi\, 2}{4}\, 0}}_{e^{j0}=1} + \underbrace{A_3[1]}_{-1} \underbrace{e^{j\frac{2\pi\, 3}{4}\, 0}}_{e^{j0}=1} \right) = 0$$

$$s^{(1)}[1] = \frac{1}{\sqrt{T}} \left(\underbrace{A_0[1]}_{+1} \underbrace{e^{j\frac{2\pi\, 0}{4}\, 1}}_{e^{j0}=+1} + \underbrace{A_1[1]}_{-1} \underbrace{e^{j\frac{2\pi\, 1}{4}\, 1}}_{e^{j\pi/2}=+j} + \underbrace{A_2[1]}_{+1} \underbrace{e^{j\frac{2\pi\, 2}{4}\, 1}}_{e^{j\pi}=-1} + \underbrace{A_3[1]}_{-1} \underbrace{e^{j\frac{2\pi\, 3}{4}\, 1}}_{e^{j3\pi/2}=-j} \right) = 0$$

$$s^{(1)}[2] = \frac{1}{\sqrt{T}} \left(\underbrace{A_0[1]}_{+1} \underbrace{e^{j\frac{2\pi\, 0}{4}\, 2}}_{e^{j0}=+1} + \underbrace{A_1[1]}_{-1} \underbrace{e^{j\frac{2\pi\, 1}{4}\, 2}}_{e^{j\pi}=-1} + \underbrace{A_2[1]}_{+1} \underbrace{e^{j\frac{2\pi\, 2}{4}\, 2}}_{e^{j2\pi}=+1} + \underbrace{A_3[1]}_{-1} \underbrace{e^{j\frac{2\pi\, 3}{4}\, 2}}_{e^{j3\pi}=-1} \right) = \frac{4}{\sqrt{T}}$$

$$s^{(1)}[3] = \frac{1}{\sqrt{T}} \left(\underbrace{A_0[1]}_{+1} \underbrace{e^{j\frac{2\pi\, 0}{4}\, 3}}_{e^{j0}=+1} + \underbrace{A_1[1]}_{-1} \underbrace{e^{j\frac{2\pi\, 1}{4}\, 3}}_{e^{j3\pi/2}=-j} + \underbrace{A_2[1]}_{+1} \underbrace{e^{j\frac{2\pi\, 2}{4}\, 3}}_{e^{j3\pi}=-1} + \underbrace{A_3[1]}_{-1} \underbrace{e^{j\frac{2\pi\, 3}{4}\, 3}}_{e^{j9\pi/2}=+j} \right) = 0$$

Third block: $s^{(2)}[m] \equiv \{s[8], s[9], s[10], s[11]\}$

$$s^{(n)}[m] = \frac{N}{\sqrt{T}} \text{IDFT}_N \{A_0[2], A_1[2], A_2[2], A_3[2]\} = \frac{N}{\sqrt{T}} \text{IDFT}_N \{-1, +3, -1, -3\}$$

$$s^{(n)}[0] = \frac{1}{\sqrt{T}} \left(\underbrace{A_0[2]}_{-1} \underbrace{e^{j\frac{2\pi\,0}{4}\,0}}_{e^{j0}=1} + \underbrace{A_1[2]}_{+3} \underbrace{e^{j\frac{2\pi\,1}{4}\,0}}_{e^{j0}=1} + \underbrace{A_2[2]}_{-1} \underbrace{e^{j\frac{2\pi\,2}{4}\,0}}_{e^{j0}=1} + \underbrace{A_3[2]}_{-3} \underbrace{e^{j\frac{2\pi\,3}{4}\,0}}_{e^{j0}=1} \right) = \frac{-2}{\sqrt{T}}$$

$$s^{(n)}[1] = \frac{1}{\sqrt{T}} \left(\underbrace{A_0[2]}_{-1} \underbrace{e^{j\frac{2\pi\,0}{4}\,1}}_{e^{j0}=+1} + \underbrace{A_1[2]}_{+3} \underbrace{e^{j\frac{2\pi\,1}{4}\,1}}_{e^{j\pi/2}=+j} + \underbrace{A_2[2]}_{-1} \underbrace{e^{j\frac{2\pi\,2}{4}\,1}}_{e^{j\pi}=-1} + \underbrace{A_3[2]}_{-3} \underbrace{e^{j\frac{2\pi\,3}{4}\,1}}_{e^{j3\pi/2}=-j} \right) = \frac{+j6}{\sqrt{T}}$$

$$s^{(n)}[2] = \frac{1}{\sqrt{T}} \left(\underbrace{A_0[2]}_{-1} \underbrace{e^{j\frac{2\pi\,0}{4}\,2}}_{e^{j0}=+1} + \underbrace{A_1[2]}_{+3} \underbrace{e^{j\frac{2\pi\,1}{4}\,2}}_{e^{j\pi}=-1} + \underbrace{A_2[2]}_{-1} \underbrace{e^{j\frac{2\pi\,2}{4}\,2}}_{e^{j2\pi}=+1} + \underbrace{A_3[2]}_{-3} \underbrace{e^{j\frac{2\pi\,3}{4}\,2}}_{e^{j3\pi}=-1} \right) = \frac{-2}{\sqrt{T}}$$

$$s^{(n)}[3] = \frac{1}{\sqrt{T}} \left(\underbrace{A_0[2]}_{-1} \underbrace{e^{j\frac{2\pi\,0}{4}\,3}}_{e^{j0}=+1} + \underbrace{A_1[2]}_{+3} \underbrace{e^{j\frac{2\pi\,1}{4}\,3}}_{e^{j3\pi/2}=-j} + \underbrace{A_2[2]}_{-1} \underbrace{e^{j\frac{2\pi\,2}{4}\,3}}_{e^{j3\pi}=-1} + \underbrace{A_3[2]}_{-3} \underbrace{e^{j\frac{2\pi\,3}{4}\,3}}_{e^{j9\pi/2}=+j} \right) = \frac{-j6}{\sqrt{T}}$$

OFDM Samples

m	0	1	2	3	4	5	6	7	8	9	10	11
$A[m]$	$+1$	-3	$+1$	$+3$	$+1$	-1	$+1$	-1	-1	$+3$	-1	-3

- Samples of each block : $s^{(n)}[m] = s[nN+m]$, $m \in \{0, 1, 2, 3\}$

$$s^{(0)}[m] = \frac{N}{\sqrt{T}} \text{IDFT}_N \{+1, -3, +1, +3\} = \left\{ \frac{+2}{\sqrt{T}}, \frac{-j6}{\sqrt{T}}, \frac{+2}{\sqrt{T}}, \frac{+j6}{\sqrt{T}} \right\}$$

$$s^{(1)}[m] = \frac{N}{\sqrt{T}} \text{IDFT}_N \{+1, -1, +1, -1\} = \left\{ 0, 0, \frac{+4}{\sqrt{T}}, 0 \right\}$$

$$s^{(2)}[m] = \frac{N}{\sqrt{T}} \text{IDFT}_N \{-1, +3, -1, -3\} = \left\{ \frac{-2}{\sqrt{T}}, \frac{+j6}{\sqrt{T}}, \frac{-2}{\sqrt{T}}, \frac{-j6}{\sqrt{T}} \right\}$$

m	0	1	2	3	4	5	6	7	8	9	10	11
$s[m]$	$\frac{+2}{\sqrt{T}}$	$\frac{-j6}{\sqrt{T}}$	$\frac{+2}{\sqrt{T}}$	$\frac{+j6}{\sqrt{T}}$	0	0	$\frac{+4}{\sqrt{T}}$	0	$\frac{-2}{\sqrt{T}}$	$\frac{+j6}{\sqrt{T}}$	$\frac{-2}{\sqrt{T}}$	$\frac{-j6}{\sqrt{T}}$