DIGITAL COMMUNICATIONS
THEORY
(Time: 60 minutes. Grade 4/10)

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Question 1

A digital communication system can use the frequency range between 200 MHz and 246 MHz. The system will employ a FDM ("*Frequency Division Multiplex*") technique for transmission, with 4 channels and a guard interval of 2 MHz between channels (the system will not use guards before the first channel and after the end of the last channel). In all the 4 channels identical modulations (16-QAM), transmitter filters (root raised cosine filters with roll-off factor $\alpha = 1$) and transmission rates will be used, being the only channel-specific parameter the carrier frequency, $\omega_{c,0}$, $\omega_{c,1}$, $\omega_{c,2}$ and $\omega_{c,3}$, respectively, in rad/s.

a) If the available bandwidth is used efficiently taking into account the required guard intervals, obtain the carrier frequency for each channel.

b) Calculate the binary rate (bits/s) for each channel and the total binary rate of the FDM system.

c) If the roll-off factor α of the transmitter filters could be modified, with the goal of increassing as much as possible the transmission rate, explain the value for α that you will choose and the corresponding value for binary transmission rate that can be achieved per channel and with the whole FDM system.

(1 point)

Question 2

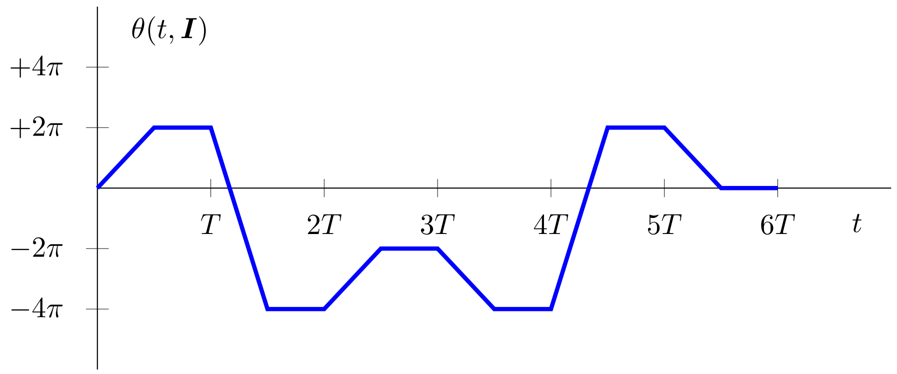
a) A communication system uses a frequency modulation with pulses using the following four frequencies

$$f_0 = 2\ \text{MHz},\ f_1 = 3.5\ \text{MHz},\ f_2 = 5\ \text{MHz},\ f_3 = 6.5\ \text{MHz}$$

  I) Explain clearly the conditions that the 4 frequencis have to satisfy in a continuous phase frequency shift keying modulation (CPFSK), assess if the 4 given frequencies satisfy these conditions, and in the case of a positive answer obtain the symbol rate and binary rate of the CPFSK system.

  II) Explain clearly the conditions that the 4 frequencis have to satisfy in a minimum shift keyng modulation (MSK), assess if the 4 given frequencies satisfy these conditions, and in the case of a positive answer obtain the symbol rate and binary rate of the MSK system.

b) A continuous phase modulation (CPM) is considered now. Specifically, a quaternary full-response CPM with normalized pulse and symbols (i.e., $I[n] \in \{\pm 1, \pm 3\}$). The following picture plots the evolution of the phase of the CPM modulation, $\theta(t, \boldsymbol{I})$ during the transmission of the first six symbols.



  I) Obtain the value for the modulation index, h.

  II) Obtain the sequence of the first six transmitted symbols.

  III) Obtain the normalized pulse $g(t)$.

(1.5 points)

Question 3

An OFDM modulation with $N = 4$ carriers is transmitted through a channel whose equivalent discrete response at time $T(N + C)$ is

$$d[m] = \delta[m] + 0.3\delta[m-1]$$

where C is the minimal length of the cyclic prefix allowing to avoid intersymbol and intercarrier interferences. First samples of the extended discrete time sequence of samples of the OFDM signal, corresponding to symbols $A_0[0]$, $A_1[0]$, $A_2[0]$, $A_3[0]$, have the folowing values when $T = 1$ is considered

$$\widetilde{s}[-1] = 2 + j2, \quad \widetilde{s}[0] = 0, \quad \widetilde{s}[1] = 2 - j2, \quad \widetilde{s}[2] = 0, \quad \widetilde{s}[3] = 2 + j2$$

a) Determine which is the lenght of the cyclic prefix that is used for transmission and explain the reason for such choice.

b) From values of the extended sequence $\widetilde{s}[m]$, obtain the values of the symbols transmitted in the first symbol interval, $A_0[0]$, $A_1[0]$, $A_2[0]$, $A_3[0]$.

c) Obtain the values for the first N samples of the signal received at discrete time given at T/N, $v[0]$, $v[1]$, $v[2]$, $v[3]$ when noise is not considered.

d) Obtain the symbol-rate observations in the first symbol interval $q_0[0]$, $q_1[0]$, $q_2[0]$, $q_3[0]$.

(1.5 points)

DIGITAL COMMUNICATIONS
EXERCISES
(Time: 120 minutes. Grade 6/10)

	Grade	
Last Name(s):	1	
First (Middle) Name:	2	
ID Number: .... Group	T	
Signature		

Exercise 1

A digital communication system transmits a 2-PAM modulation with normalized symbols ($A[n] \in \{\pm 1\}$) through the following equivalent discrete channel

$$p[n] = 0.3\ \delta[n] + 0.5\ \delta[n-1] + 0.1\ \delta[n-2]$$

with a discrete time noise sequence $z[n]$ white and Gaussian with variance $\sigma_z^2 = 0.01$.

a) If the receiver uses a memoriless symbol-by-symbol detector

 i) Obtain the optimal delay for decision, explaining how this value is obtained.

 ii) Calculate the probability of error achieved using this detector.

b) If the receiver uses a channel equalizer

 i) Obtain the MMSE equalizer with 5 coefficients and a delay $d = 3$ (state the equation system that has to be solved, providing the numerical values of all involved terms, but it is not necessary to solve the system).

 ii) Obtain the optimal delay and calculate the approximate probability of error of the receiver if the following equalizer of 2 coefficients is used

$$w[n] = -\delta[n] + 2\ \delta[n-1].$$

c) If the receiver uses a maximum likelihood sequence detector

 i) Plot the trellis diagram of the detector.

 ii) Calculate the approximate probability of error obtained using this detector.

 iii) Applying the optimal detection algorithm, obtain the maximum likelihood solution for $A[0]$, $A[1]$ and $A[2]$ if the system transmits data block of three symbols beteen headers of the necessary lenght to reset the system using $A[n] = +1$ for all symbols in the header. Observations at the demodulator output are

n	0	1	2	3	4
$q[n]$	0.5	0.4	0	-0.1	0.8

 REMARK: clear evidence of the application of the optimal decoding algorithm has to be provided.

(3 points)

Exercise 2

The parity check matrix of a linear block code is

$$\boldsymbol{H} = \left[\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & a & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & b \\ 0 & 0 & 1 & 0 & 0 & c & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right]$$

a) Determine the number of bits of the uncoded words, k, the number of bits of the coded works, n, the code rate, and the number of words of the code.

b) Determine if the code is systematic or not, explaining clearly the response.

c) Obtain the values for a, b and c that allow to obtan the best performance (the procedure that is used to obtain these values has to be clearly explained).

d) Obtain the syndrome table allowing to obtain the best performance and decode by using the symdrome based decoding technique the following received words

$$\mathbf{r}_1 = [1001100] \qquad \mathbf{r}_2 = [0101000]$$

Each step of the decoding procedure has to be detailed. If the previous section has not been solved, use for instance $a = b = c = 1$ for this section.

e) Obtain the probability of error obtained with this code when the bit error rate of the system transmitting the coded bits is ε and the decoder works with hard decisions.

(3 puntos)