DIGITAL COMMUNICATIONS
PART A
(Time: 60 minutes. Points 4/10)

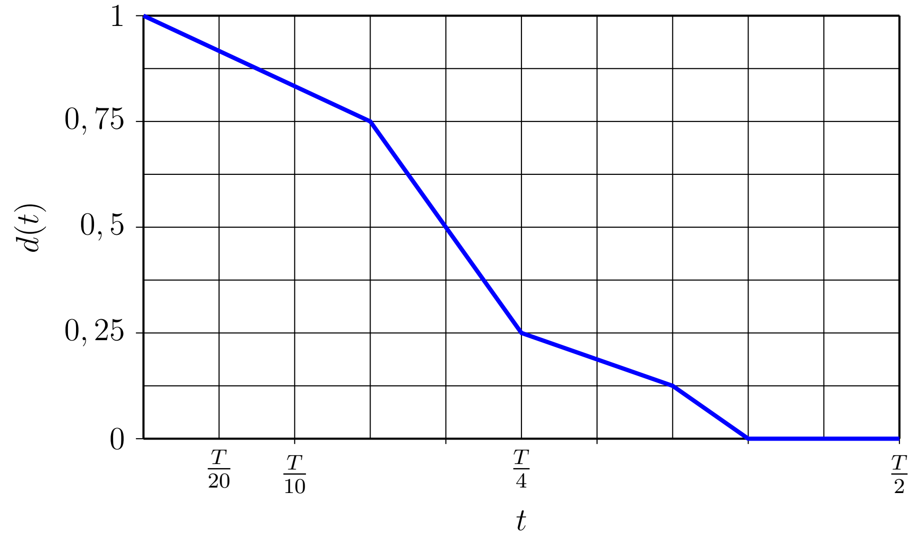
Last Name(s): ....
First (Middle) Name: ....
ID number: .... Group ....
Signature

Grades	
1	
2	
3	
T	

Exercise 1

A communication system uses a discrete time OFDM modulation with $N = 4$ carriers and with symbol length T.

Joint transmitter, channel and receiver response $d(t)$ is the one given in the picture (in a real case, this joint response depends on the reconstruction rate of the transmitter filter; for the sake of simplicity, assume that this is the response for any rate).



1. If the preliminary system does not use cyclic prefic, determine if inter-symbol interference (ISI) and/or inter-carrier interference (ICI) will be present. In the case ISI/ICI are present, design an alternative systen to avoid them being the more efficient as possible.

2. Obtain the equivalent discrete channels $p_{k,i}[n]$ for the system designed in previous section.

(1 point)

Exercise 2

A digital communication system transmits at a binary rate $R_b = 10$ kbits/s and has assigned the frequency band between 5 kHz and 10 kHz. Transmitter and receiver use normalized root raised cosine filters with roll-off factor α. Constellation is a M-QAM with normalized levels, and transmitted data sequence $A[n]$ is white.

a) Obtain the carrier frequency, the power of the modulated signal, the bandwidth of the modulated signal and the constellation order, M, if roll-off factor is $\alpha = 0$.

b) Repeat previous section if now roll-off factor is $\alpha = 0.75$.

c) Given that $\alpha = 0$, now the channel has response

$$h(t) = \text{sinc}^2\left(10^4\, t\right)$$

Obtain the equivalent discrete channel, in the time domain or in the frequency domain, and given this equivalent discrete channel discuss about if intersymbol interference will appear during transmission.

(1.5 points)

Exercise 3

Several systems with different angle modulations will be analyzed.

a) A 4-ary mimimum shift keying (MSK) modulation is employed to tramsmit at binary rate R_b=2 Mbits/s, with the constraint that all frequencies associated to the modulation pulses have to satisfy $\omega_i \geq 3\pi$ Mrad/s (or $f_i \geq 1.5$ MHz) for $i \in \{0, 1, 2, 3\}$. Obtain the 4 frequencies of the system with the purpose of having the lowest possible values.

b) Repeat the previous question if now a continuous phase frequency shift keying (CPFSK) modulation is used.

c) Consider now a differential phase shift keying modulation using a QPSK constellation with normalized levels

$$\boldsymbol{a}_0 = \begin{bmatrix} +1 \\ +1 \end{bmatrix}, \; \boldsymbol{a}_1 = \begin{bmatrix} -1 \\ +1 \end{bmatrix}, \; \boldsymbol{a}_2 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}, \; \boldsymbol{a}_3 = \begin{bmatrix} +1 \\ -1 \end{bmatrix}.$$

Design the binary assignment of the DPSK system and, assuming that previous symbol (reference symbol) is $A[-1] = \boldsymbol{a}_0$, obtain the symbol sequence $A[n]$ that is produced by the following bit sequence

m	0	1	2	3	4	5	6	7
$B[m]$	0	1	1	1	1	0	0	0

(1,5 points)

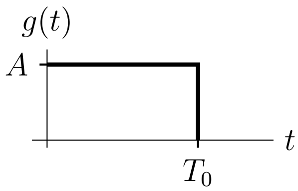
DIGITAL COMMUNICATIONS
PART B
(Time: 120 minutes. Points 6/10)

Last Name(s): ....
First (Middle) Name: ....
ID number: .... Group ....
Signature

Grades	
4	
5	
T	

Exercise 4

A digital baseband communication system transmits a 2-PAM constellation at a binary rate $R_b = 1$ kbits/s. The white sequence of 2-PAM equiprobable symbols, denoted as $A[n]$, is filtered to obtain sequence $B[n] = A[n] * h_c[n]$, where $h_c[n] = \delta[n] + 0.3\delta[n-1]$. Finally, sequence $B[n]$ is used at the input of the transmitter filter to generate the modulated baseband signal $s(t)$. Thermal noise has power spectral density $N_0/2$ with $N_0 = 0.1$. Transmitter filter is given in the picture



a) Assuming an ideal channel ($h(t) = \delta(t)$) and that a matched filter is used at the receiver ($f(t) = g(-t)$), obtain the value (or values) for T_0 allowing a communication free of intersymbol interference (ISI).

b) Obtain the power spectral density of $s(t)$ for $T_0 = \frac{1}{2R_b}$. Plot this power spectral density (approximately), properly labeling both axes.

c) If now the channel is not ideal and equivalent discrete channel is $p[n] = \delta[n] + 0.75\delta[n-1]$, obtain the optimal delay and decision regions to detect sequence $A[n]$ from observations $q[n]$ if a memoryless symbol-by-symbol detector is used. Assume that SNR is relatively high.

d) For the equivalent discrete channel of previous section, design the channel equalizer with 3 coefficients and MMSE criterion for a delay $d = 2$ designed to recover $A[n]$ from $q[n]$.

<u>REMARK</u>: It is not necessary to solve the system, but all numerical values involved in the system to be solved have to be provided.

(3 points)

Exercise 5

a) A linear block code has the dictionary given in these tables

$\boldsymbol{b}_i$	$\boldsymbol{c}_i$
0 0 0 0	0 0 0 0 0 0 0
0 0 0 1	0 1 1 1 1 0 0
0 0 1 0	1 0 1 1 0 1 0
0 0 1 1	1 1 0 0 1 1 0
0 1 0 0	1 1 1 0 0 0 0
0 1 0 1	1 0 0 1 1 0 0
0 1 1 0	0 1 0 1 0 1 0
0 1 1 1	0 0 1 0 1 1 0

$\boldsymbol{b}_i$	$\boldsymbol{c}_i$
1 0 0 0	1 1 1 1 1 1 1
1 0 0 1	1 0 0 0 0 1 1
1 0 1 0	0 1 0 0 1 0 1
1 0 1 1	0 0 1 1 0 0 1
1 1 0 0	0 0 0 1 1 1 1
1 1 0 1	0 1 1 0 0 1 1
1 1 1 0	1 0 1 0 1 0 1
1 1 1 1	1 1 0 1 0 0 1

I) Obtain the following parameters for that code:
   - Coding rate and generating matrix.
   - Minimum Hamming distance, explaing clearly how it was obtained, and the number of errors that the code is able to correct working with hard output.
   - Discuss if the code is perfect or not, explaing clearly the reason.

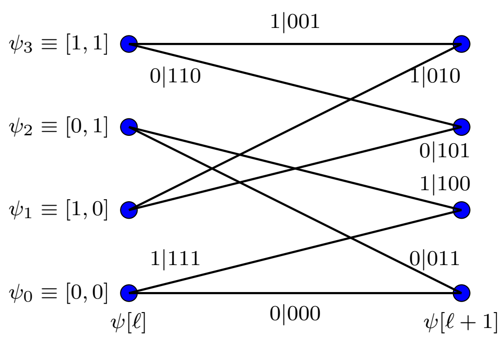
II) Obtain the parity check matrix and the syndrome table.

III) Using the syndrome based decoding technique, enumerating each step, decode the following received word

$$\boldsymbol{r} = 0\ 1\ 1\ 1\ 0\ 1\ 1$$

b) Two convolutional encoders are available. For the first one, its generating matrix is know, and for the second one the trellis diagram is provided, which are the ones shown below

$$\boldsymbol{G} = \begin{bmatrix} 1+D^2 & D & 1 \\ D & 1+D & 1 \end{bmatrix}$$



I) For the first encoder, obtain the schematic representation and plot the trellis diagram partialy, drawing only the branches going out of the states $\psi[\ell]$ all zeros and all ones, respectively, and arriving at the corresponding states $\psi[\ell+1]$.

II) For the second encoder, obtain the schematic representation and its generating matrix.

III) For the second encoder, decode the bits $B^{(0)}[0]$, $B^{(0)}[1]$ and $B^{(0)}[2]$ assuming that headers of zeros are transmitted before and after these 3 bits, if the sequence of received bits is

m	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$R[m]$	0	1	0	1	1	1	1	0	0	1	0	1	0	0	1

REMARK: clear evidence of the application of the optimal algorithm must be provided

(3 puntos)