DIGITAL COMMUNICATIONS
PART A
(Time: 60 minutes. Points 4/10)

Last Name(s): .... First (Middle) Name: .... ID number: .... Group .... Signature	Grades	
	1	
	2	
	3	
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Exercise 1

a) A phase modulation employs a 8-PSK constellation with symbols

$$\mathbf{a}_0 = \begin{bmatrix} +1 \\ 0 \end{bmatrix} \quad \mathbf{a}_1 = \begin{bmatrix} +a \\ +a \end{bmatrix} \quad \mathbf{a}_2 = \begin{bmatrix} 0 \\ +1 \end{bmatrix} \quad \mathbf{a}_3 = \begin{bmatrix} -a \\ +a \end{bmatrix} \quad \mathbf{a}_4 = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \quad \mathbf{a}_5 = \begin{bmatrix} -a \\ -a \end{bmatrix} \quad \mathbf{a}_6 = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \quad \mathbf{a}_7 = \begin{bmatrix} +a \\ -a \end{bmatrix}$$

where $a = \frac{1}{\sqrt{2}}$. Provide an appropriate binary assignment in the following cases:

i) A conventional phase shift keying (PSK) modulation is used.

ii) A differential phase shift keying (DPSK) modulation is used.

b) In a continuous phase modulation (CPM), explain the difference between a partial-response and a full-response modulation, identifying the differential feature for each variant, and provide an illustrative example for each one of them.

c) Explain how phase continuity is obtained, and write the conditions that the frequencies of the different pulses that are used have to satisfy in the following frequency modulations:

i) Continuous phase frequency shift keying (CPFSK) modulation.

ii) Minimum shift keying (MSK) modulation.

(1.5 points)

Exercise 2

A baseband communication system uses a direct sequence spread spectrum modulation with spreading factor $N = 3$ and spreading sequence

$$x[m] = +\delta[m] - \delta[m-1] + \delta[m-2]$$

to transmit at a binary rate of 1 kbit/s using an 8-PSK modulation with normalized levels, with the transmitted sequence being white. Transmitter filter at chip rate, $g_c(t)$, is a root raised cosine at chip rate with roll-off factor α, receiver filter is matched to the transmitter filter, the channel impulse response is

$$h(t) = \delta(t) - \frac{1}{3}\delta(t - 7 \times 10^{-3})$$

and thermal noise is white and Gaussian with power spectral density $N_0/2$.

a) Obtain the analytical expression of the power spectral density of the modulated baseband signal, $s(t)$, for a generic value of the roll-off factor, and plot this power spectral density and provide the bandwidth of the modulated signal in the following case: $\alpha = 0$.

   REMARK: In the figure plotting the power spectral density, axes have to be properly labeled, including the appropriate numerical values.

(1 point)

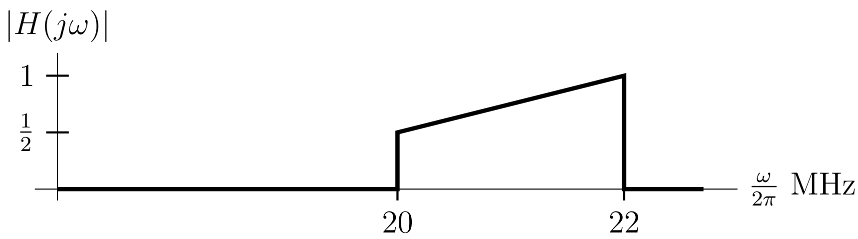
Exercise 3

A digital communication system has assigned the frequency band between 20 MHz and 22 MHz, uses matched filters in transmitter and receiver, and a 64-QAM constellation with normalized levels. Additive noise in the channel is white and Gaussian, with power spectral density $N_0/2$.

a) If the channel response in the assigned band is ideal, design the normalized transmitter filter and the carrier frequency to obtain the maximum possible binary transmission rate without intersymbol interference (ISI), and using this filter obtain the maximum binary rate, and discuss if the sampled noise at the output of the receiver, $z[n]$, is or not white explaining clearly the reason.

b) Design now the normalized transmitter filter and the carrier frequency to transmit without ISI at a binary rate of 10 Mbits/s, using the whole available bandwidth, and assuming again an ideal channel response.

c) If the channel response is the one shown in the figure, design the transmitter filter to transmit without ISI, and demonstrate if the sampled noise $z[n]$ is or not white in this case.



(1.5 points)

DIGITAL COMMUNICATIONS
PART B
(Time: 120 minutes. Points 6/10)

	Grades	
Last Name(s):	4	
First (Middle) Name:	5	
ID number: .... Group	T	
Signature		

Exercise 4

A baseband digital communication system has the following equivalent discrete channel

$$p[n] = \frac{1}{4}\,\delta[n] - 2\,\delta[n-1] + \frac{1}{4}\,\delta[n-2]$$

and sampled noise at the output of the demodulator is white and Gaussian with variance $\sigma_z^2 = 0.01$.

a) In this case a memoryless symbol-by-symbol detector, designed to obtain the best possible performance is used.

   i) If the transmitted constellation is a 4-PAM with normalized levels, design the optimal memoryless symbol-by-symbol detector providing clearly all parameters (delay, decision regions,...).

   ii) If the transmitted constellation is a 2-PAM with normalized levels, obtain the exact probability of error of the system.

b) Now a channel equalizer, without constraints in the number of coefficients, is employed.

   i) Design the equalizer using the zero forcing (ZF) criterion, and obtain the probability of error if the transmitted constellation is a 4-PAM constellation with normalized levels.

   ii) Explain how the optimal delay is obtained for this kind of equalizers.

c) Finally, a channel equalizer with 3 coefficients is used, with coefficients

n	0	1	2
$w[n]$	-0.1	-0.5	-0.1

   i) Obtain the optimal delay for the decision with this receiver, explaining clearly how it was obtained.

   ii) Estimate the probability of error achieved with this receiver, if the transmitted constellation is a 4-PAM with normalized levels.

(3 points)

Exercise 5

a) A systematic linear block code has a syndrome table that is partially shown below

e	**s**
0 0 0 0 1	0 0 1
1 0 0 0 1	1 1 1
1 1 0 0 0	1 0 1

i) Obtain the parity check matrix and the complete syndrome table.

ii) Obtain the following parameters for the code:

- Coding rate.
- Generating matrix.
- Minimum distance of the code, explaining how this distance is obtained.
- Number of errors that the code is able to detect and to correct working with hard output.
- Explain if the code is perfect or not, explaining clearly the reason.
- Obtain the probability of error for this code working in a system that transmits a 2-PAM constellation through an ideal channel with Gaussian noise with power spectral density $N_0/2$.

iii) Using the syndrome based decoding technique, and itemizing every step, decode the following received word

$$\mathbf{r} = 1\ 1\ 1\ 1\ 1$$

b) A convolutional encoder has the following generating matrix

$$\mathbf{G}(D) = [1 + D + D^2 \qquad 1 + D^2 \qquad 1 + D]$$

i) Obtain the schematic representation and the trellis diagram of the encoder.

ii) Obtain the performance of the convolutional encoder working in a system that transmits a 2-PAM constellation through an ideal channel with Gaussian noise with power spectral density $N_0/2$.

iii) Decode the bits $B^{(0)}[0]$, $B^{(0)}[1]$ and $B^{(0)}[2]$ applying the optimal algorithm and assuming that headers with an appropriate number of zeros are transmitted before and after those 3 data bits, if the received binary sequence (hard output) is

m	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$R[m]$	0	1	0	1	1	1	0	0	1	1	0	1	1	0	0

REMARK: clear evidence of the application of the optimal algorithm has to be provided

(3 points)