DIGITAL COMMUNICATIONS

PART A

(Time: 60 minutes. Points 4/10)

Last Name(s): ....

First (Middle) Name: ....

ID number: .... Group ....

Signature

Grades	
1	
2	
3	
T	

Exercise 1

A frequency modulation employs 8 frequencies to transmit digital information. Discuss if the 8 frequencies are valid for a CPFSK modulation and/or for a MSK modulation, explaining clearly the reason for each modulation, and in the case of a positive answer obtain the symbol rate and the binary rate for each modulation when

a) The 8 frequencies are:

f_1 (kHz)	f_2 (kHz)	f_3 (kHz)	f_4 (kHz)	f_5 (kHz)	f_6 (kHz)	f_7 (kHz)	f_8 (kHz)
1500	1600	1700	1800	1900	2000	2100	2200

b) The 8 frequencies are:

f_1 (kHz)	f_2 (kHz)	f_3 (kHz)	f_4 (kHz)	f_5 (kHz)	f_6 (kHz)	f_7 (kHz)	f_8 (kHz)
1500	1700	1900	2100	2300	2500	2700	2900

(1 point)

Exercise 2

The following sequences are known

n	0	1	2	3
$A[n]$	$+1$	-3	$+1$	-1

m	0	1	2	3
$v[m]$	$+1.1$	-0.9	-0.8	$+0.7$

a) A direct sequence spread spectrum modulation with spreading factor $N = 4$, spreading sequence

m	0	1	2	3
$x[m]$	$+1$	-1	-1	$+1$

and carrier frequency $f_c = 1$ MHz is employed. Transmitter filter at chip rate is a normalized root-raised cosine filter with roll-off factor $\alpha = 0.2$.

i) Obtain the samples at chip rate associated to the transmission of data sequence $A[n]$ at a symbol rate of 5 kbauds, making explicit the discret instant associated to each sample (similarly as in the tables given above), and obtain the bandwidth of the modulated signal.

ii) Compute the observations at symbol rate $q[n]$ associated to the processing of the observations at chip rate obtained at the ouput of the receiver filter, $v[m]$, making explicit the discret instant n associated to each observation.

b) Now an OFDM modulation with 4 carriers is used to transmit data sequence $A[n]$ at a total symbol rate of 4 bauds with a carrier frequency $f_c = 1$ MHz.

i) Without cyclic prefix, compute the value of the samples of the transmitted signal at T/N associated to data sequence $A[n]$, making explicit the discret instant associated to each sample (similarly as in the tables given above), and obtain the bandwidth of the modulated signal.

ii) With a cyclic prefix of length $C = 1$ sample, compute the value of the samples of the transmitted signal at $T/(N + C)$ associated to data sequence $A[n]$, making explicit the discret instant associated to each sample (similarly as in the tables given above), and obtain the bandwidth of the modulated signal.

(1 point)

Exercise 3

A digital communication system has allotted the frequency band between 0 Hz and 5 kHz. Receiver filter is matched to the transmitter. Additive noise is white and Gaussian, with power spectral density $N_0/2$, and the transmitted data sequence is white.

a) If channel response in the allotted band is ideal, design the normalized transmitted filter to transmit at the maximum possible symbol rate without inter-symbol interference (ISI), compute the value of the maximum symbol rate, and discuss if sampled noise, $z[n]$, is or not white, explaining clearly the reason.

b) Design the normalized transmitter filter and the constellation to transmit without ISI at a binary rate of 32 kbits/s, employing the whole available bandwidth, and assuming again an ideal channel.

c) Plot the power spectral density of the modulated signal generated by the system designed in the previous section (appropriate labels, including relevant numerical values, have to be provided for both axes), and compute the power of that signal.

d) The channel now has the following frequency response

$$H(j\omega) = \begin{cases} 1 - \frac{|\omega|}{2\pi\times 10^4} & \text{if } |\omega| \leq 2\pi \times 10^4 \text{ rad/s} \\ 0 & \text{if } |\omega| > 2\pi \times 10^4 \text{ rad/s} \end{cases}$$

Design the transmitter filter to allow an ISI-free transmission.
REMARK: remember that the system can use only the frequency band between 0 and 5 kHz.

(2 points)

DIGITAL COMMUNICATIONS
PART B
(Time: 120 minutes. Points 6/10)

	Grades	
Last Name(s):	4	
First (Middle) Name:	5	
ID number: .... Group	T	
Signature		

Exercise 4

A baseband digital communication system has the following equivalent discrete channel

$$p[n] = \delta[n] + \frac{1}{4}\,\delta[n-1] - 4\,\delta[n-2]$$

sampled noise $z[n]$ is white, Gaussian with variance $\sigma_z^2 = 0.2$, and M-PAM constellations with normalized levels are used.

a) In this section, a memoryless symbol-by-symbol detector is used

  i) If the transmitted constellation is a 4-PAM, design the optimal detector providing all parameters (delay, decision regions,...), and compute decisions $\hat{A}[n]$ for $n \in \{0, 1, 2, 3\}$ if observations are:

n	0	1	2	3	4	5	6	7	8	9
$q[n]$	+0.7	+3.8	+1.3	−7.4	−12.1	+2.5	−15.3	+17.4	−0.1	+0.25

  ii) For a 2-PAM constellation, design the optimal detector providing all parameters, and compute the exact probability of error of that detector.

b) Now a channel equalizar with 3 coefficients is used and a 4-PAM is transmitted.

  i) Obtain the coefficients of the equalizer using the MMSE design criterion for a delay $d = 2$.

  ii) Obtain the optimal delay for decision (explaining how this value is obtained) and compute the probability of error if equalizer is $w[n] = -0.3\,\delta[n] + 0.1\,\delta[n-1] - 0.2\,\delta[n-2]$.

c) Finally, a maximum likelihood sequence detector is used when a 2-PAM is transmitted. All symbols included in the necessary cyclic header have the value $A[n] = +1$.

  i) Obtain the trellis diagram for this detector.

  ii) Estimate the probability of error obtained with this detector.

(3 points)

Exercise 5

a) A linear block code has the following parity check matrix

$$\mathbf{H} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

I) Obtain a generating matrix compatible with $\mathbf{H}$ in three cases:
   - The code is systematic at the beginning (generating matrix $\mathbf{G}_1$)
   - The code is systematic at the end (generating matrix $\mathbf{G}_2$)
   - The code is non systematic (generating matrix $\mathbf{G}_3$)

II) For the code that is systematic at the beginning (generating matrix $\mathbf{G}_1$) obtain
   - Coding rate and dictionary
   - Minimum distance of the code, explaining how this distance is obtained.
   - Number of errors that the code is able to detect and to correct working with hard output.
   - Explain if the code is perfect or not, explaining clearly the reason.

III) Again for the code with $\mathbf{G}_1$, obtain the syndrome table and using the syndrome based decoding technique (itemizing every step) decode the following received word

$$\mathbf{r} = 1\ 1\ 1\ 1\ 1$$

b) A convolutional encoder has the following generating matrix

$$\mathbf{G}(D) = [1 + D + D^2 \qquad 1 + D^2 \qquad 1 + D]$$

I) Obtain the schematic representation of the encoder.

II) Obtain the trellis diagram of the encoder.

III) Encode the following binary sequence of 3 data bits

m	0	1	2
$B[m]$	1	1	1

assuming that headers with an appropriate number of zeros are transmitted before and after those 3 data bits. Encoded data sequence $C[m]$ must be provided from $m = 0$ up to the last relevant value for m (last encoded bit that depends on the given data bits).

IV) Decode the bits $B[0]$, $B[1]$ and $B[2]$ applying the optimal algorithm and assuming that headers with an appropriate number of zeros are transmitted before and after those 3 data bits, if the received binary sequence (hard output) is

m	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
$R[m]$	0	1	0	1	1	1	0	0	1	1	0	1	1	0	0	1	0	1	0	1	0

<u>REMARK</u>: clear evidence of the application of the optimal algorithm has to be provided

(3 points)