**DIGITAL COMMUNICATIONS**
PART A
(Time: 60 minutes. Points 4/10)

| Last Name(s): ............ | Grades | |
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| First (Middle) Name: ............ | 1 | |
| ID number: ............ Group ............ | 2 | |
| Signature | T | |

# Exercise 1

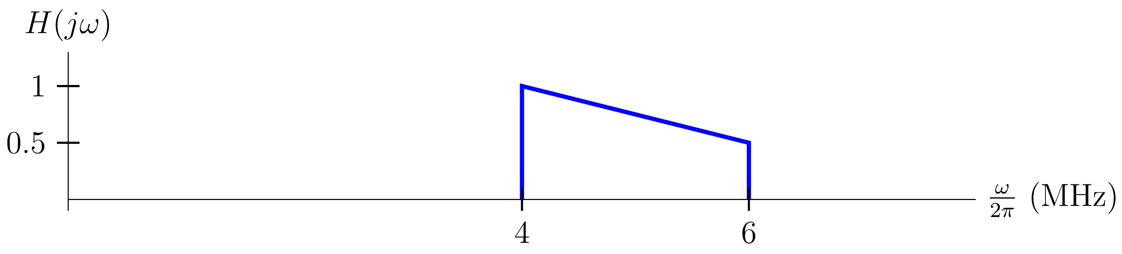
A digital communication system has assigned the frequency band between 4 MHz and 6 MHz, using root-raised cosine filters in both the transmitter and the receiver. The constellation is a $M$-QAM with normalized levels, and thermal noise is white, Gaussian, with power spectral density $N_0/2$.

a) If the channel response is ideal in the assigned band:

   I) Compute the maximum binary rate (in bits/s), and provide the values for the carrier frequency and for the parameters of the transmitter and receiver filters that allow to obtain that rate, if a 64-QAM is transmitted.

   II) Demonstrate if intersymbol interference exists during the transmission, compute the power spectral density of the sampled noise $z[n]$, and discuss if this noise is white.

b) If the frequency response of the channel in the assigned band is the one given in the picture:




   I) Provide the values of the carrier frequency, the constellation order $M$, and the parameters of the transmitter and receiver filters, which are necessary to transmit at a binary rate of 7 Mbits/s using the whole available bandwidth.

   II) Compute the power spectral density of the sampled noise $z[n]$, and discuss if this noise is white.

(2 points)

# Exercise 2

There are only three different kinds of perfect linear block codes: repetition codes, Hamming codes and the Golay code.

a) Explain the condition that a code has to satisfy to be perfect as a function of the general size of the code, ($k$ and $n$), and the number of errors that the code corrects ($t$), and explain the implication that it has in the syndrome table of the code.

b) Obtain the generator and the parity check matrices for a repetition code (1,5).

c) Obtain the generator and the parity check matrices for a Hamming code (4,7), systematic by the beginning.

d) For the repetition code (1,5), obtain the syndrome table, and decode using the syndrome based decoding technique (detailing every step of the procedure) the following received word $\mathbf{r} = 10110$.

(2 points)

**DIGITAL COMMUNICATIONS**
PART B
(Time: 120 minutes. Points 6/10)

| | Grades | |
|---|---|---|
| Last Name(s): ................................................ | 3 | |
| First (Middle) Name: ........................................ | 4 | |
| ID number: ............................ Group .................. | T | |
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# Exercise 3

A baseband digital communication system has the following equivalent discrete channel

$$p[n] = \frac{1}{5}\,\delta[n] - \delta[n-1] + \frac{1}{5}\,\delta[n-2]$$

the sampled noise at the output of the demodulator is white and Gaussian with variance $\sigma_z^2 = 0.2$, and 2-PAM constellations with normalized levels are used. The observations at the output of the demodulator are:

| $n$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $q[n]$ | $-0.7$ | $+0.4$ | $-1.1$ | $-0.7$ | $-0.7$ | $+1.2$ | $-0.2$ | $+0.6$ | $+1.2$ | $+0.7$ | $+0.3$ |

a) In this section a memoryless symbol-by-symbol detector is used.

   I) Design the optimal detector making explicit its characteristics (delay, decision regions,...), and obtain the decisions $\hat{A}[n]$ for $n \in \{0, 1, 2, 3\}$.

   II) Compute the exact probability of error for this receiver.

b) Now a channel equalizer, designed without constraints in the number of coefficients, is used.

   I) Obtain the equalizer with the MMSE criterion, and explain how the optimal delay is obtained for this kind of equalizer.

   II) Obtain the approximated probability of error of an equalizer designed with the ZF criterion, and discuss if it is better or worse for the MMSE equalizer (it is not necessary to obtain the probability of error for the MMSE equalizer).

c) Finally, a maximum likelihood sequence detector is used. All the symbols that are necessary in the periodical header take the value $A[n] = +1$.

   I) Obtain the trellis diagram of the system.

   II) If the length of the data sequence between headers is $L = 3$, obtain $\hat{A}[0]$, $\hat{A}[1]$ and $\hat{A}[2]$ by applying the optimal decoding algorithm (REMARK: clear evidence of the application of the algorithm must be provided).

(3 points)

# Exercise 4

A communication system transmits with a carrier frequency of 250 MHz at a binary rate of 8 Mbits/s. Although it is not a typical constellation for a bandpass system, a 4-PAM constellation with normalized levels is considered to simplify the computations. The initial symbols of the data sequence that is transmitted are

| $m$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $A[m]$ | $+1$ | $+3$ | $-1$ | $+3$ | $-1$ | $-3$ | $+1$ | $-3$ |

a) In this section an OFDM modulation with 4 carriers is used.

i) Explain, for a general system, how the intersymbol interference and the intercarrier interference can be avoided in an OFDM modulation (the explanation has to be precise, defining all involved terms in detail).

ii) If the cyclic prefix is NOT used, compute the discrete-time sequence of samples of the modulated signal (all the samples associated to the transmission of the data sequence given above have to be provided, making explicit the discrete instant associated to each sample), and obtain the bandwidth of the modulated signal.

iii) If a cyclic prefix of 2 samples is used, compute the discrete-time sequence of samples of the modulated signal (all the samples associated to the transmission of the data sequence given above have to be provided, making explicit the discrete instant associated to each sample), and obtain the bandwidth of the modulated signal.

b) Now, a direct sequence spread spectrum modulation is used, with spreading factor $N = 4$ and spreading sequence

| $m$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $x[m]$ | $+1$ | $-1$ | $-1$ | $+1$ |

The transmitter filter at chip rate is a root-raised cosine filter with roll-off factor $\alpha = 0.25$.

i) Compute the 8 initial samples of the discrete-time sequence at chip rate $s[m]$ (making explicit the discrete instant associated to each sample) that are associated to the transmission of the above data sequence.

ii) If the equivalent discrete channel at chip rate is $d[m] = \delta[m] + \frac{1}{2}\delta[m-2]$, and if the noise is considered negligible, obtain the second observation at symbol rate, $q[1]$, explaining clearly how this value is obtained.

iii) For the same equivalent discrete channel at chip rate, compute the equivalent discrete channel at symbol rate.

(3 points)