DIGITAL COMMUNICATIONS

PART A

(Time: 90 minutes. Points 5/10)

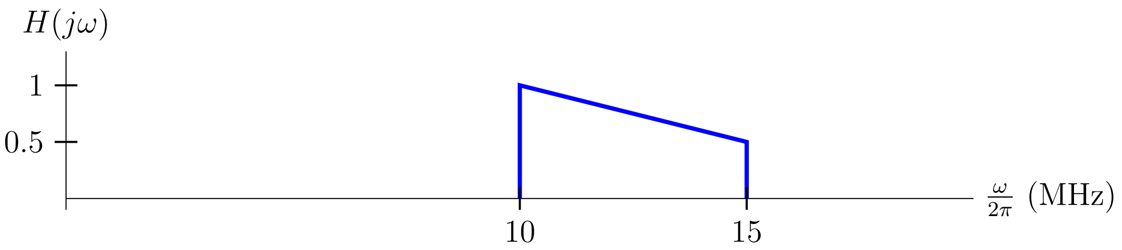
Last Name(s): ....
First (Middle) Name: ....
ID number: .... Group ....
Signature

Grades

1	
2	
T	

Exercise 1

A digital communication system transmits at a binary rate of 16 Mbits/s in the frequency band between 10 MHz and 15 MHz using matched filters in transmitter and receiver. The constellation is a M-QAM with normalized levels, and the thermal noise is white, Gaussian, with power spectral density $N_0/2$. The frequency response of the channel in the assigned band is given in the figure:



a) In this case the transmiter filter is a root-raised cosine.

i) Obtain the parameters of the transmitter filter, the order of the constellation and the carrier frequency to transmit at the required bit rate using the whole available bandwidth.

ii) With that filter, plot, appropriately labeling both axes, the joint response $P(j\omega)$, obtain the equivalent discrete channel (in time or in frequency), and discuss whether or not there is inter symbol interference (ISI).

iii) Obtain the power spectral density of the sampled noise at the demodulator output, $z[n]$, and discuss whether or not this noise is white.

b) Now a transmission without inter symbol interference (ISI) is required, using all the available bandwidth.

i) Design the transmitter filter, in time or in frequency.

ii) With that filter, plot the joint response $P(j\omega)$, and obtain the equivalent discrete channel, both in time and in frequency.

iii) Obtain the power spectral density of noise $z[n]$, and discuss whether or not this noise is white.

(2 points)

Exercise 2

A communications system transmits a 4-PAM constellation at a binary rate of 2 Mbps/s. The initial symbols of the sequence are

n	0	1	2	3	4	5	6	7
$A[n]$ or $I[n]$	-3	$+1$	-1	$+3$	-3	$+1$	-1	$+3$

a) If the modulation is a direct sequence spread spectrum modulation with spreading factor 4 and spreading sequence

m	0	1	2	3
$x[m]$	$+1$	-1	-1	$+1$

   I) Calculate the first 8 samples at chip rate, $s[m]$ for $m \in \{0, 1, \cdots, 7\}$, needed to transmit the given $A[n]$ sequence.

   II) If the transmitter filter at chip rate is a root-raised cosine with a roll-off factor $\alpha = 0.2$, calculate the bandwidth of the transmited signal in a baseband transmission.

b) Now the modulation is an OFDM with 4 carriers and a carrier frequency of 200 MHz.

   I) Calculate the discrete time samples of the OFDM signal corresponding to the transmission of the first 8 symbols of $A[n]$ if no cyclic prefix is used (clearly indicate the discrete instant associated to each sample, similarly to the tables that are shown above).

   II) Calculate the discrete time samples of the OFDM signal if now a cyclic prefix of lengh 2 is used (indicate the discrete instant associated to each sample).

   III) Calculate the bandwidth of the modulated signal in the two previous cases.

c) In this case a CPM modulation with modulation index $h = 2$ is used, with the following transmitter filter

$$g(t) = \begin{cases} A\,t, & \text{si } 0 \leq t < 10^{-6} \text{ sec.} \\ 0, & \text{in other case} \end{cases}$$

   I) Calculate the value of the constant A and say if it is a full response or partial response modulation, clearly explaining the difference between these two variants.

   II) Plot the phase tree for two symbol intervals.

(3 points)

DIGITAL COMMUNICATIONS
PART B
(Time: 90 minutes. Points 5/10)

	Grades	
Last Name(s):	3	
First (Middle) Name:	4	
ID number: .... Group	T	
Signature		

Exercise 3

A baseband digital communications system has the following equivalent discrete channel

$$p[n] = \delta[n] - 2\,\delta[n-2]$$

the noise sampled at the output of the demodulator is white and Gaussian with variance $\sigma_z^2 = 0.2$, and a 2-PAM constellation with normalized levels is used. The observations at the output of the demodulator are

n	0	1	2	3	4	5	6	7	8	9	10
$q[n]$	-0.7	$+0.4$	-1.1	-0.7	-0.7	$+1.2$	-0.2	$+0.6$	$+1.2$	$+0.7$	$+0.3$

a) In this section a memoryless symbol-by-symbol detector is used.

   I) Design the optimum symbol-by-symbol detector clearly indicating all its characteristics (delay and decision regions), and obtain the decisions $\hat{A}[n]$ for $n \in \{0, 1, 2, 3\}$.

   II) Calculate the exact probability of error that is obtained with this detector.

b) Now a linear channel equalizer designed without constraints in the number of coefficients is used.

   I) Obtain the equalizer with the MMSE design criterion, and explain how the optimal delay is obtained for this kind of equalizer.

   II) Calculate the approximated probability of error for this equalizer.

c) Now an equalizer with the following 3 coefficients is employed

n	0	1	2
$w[n]$	-0.4	0	0.1

   I) Calculate the approximated probability of error for this equalizer.

(2.5 points)

Exercise 4

a) A linear block code has the following generator matrix

$$\mathbf{G} = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

I) Obtain the following parameters for this code:

- Coding rate.
- Minimum Hamming distance, explaining clearly how it is obtained, and the number of errors that the code is able of detecting and correcting working with hard output.
- Discuss if this is a perfect code, explaining clearly the reasons.

II) Obtain the parity check matrix, the syndrome table, and using the syndrome-based decoding technique (detailing each step) decode the following received word

$$\mathbf{r} = 0\ 1\ 1\ 1\ 1\ 0$$

b) Now two convolutional codes are available. For the first one, its generator matrix is known, and for the second one its trellis diagram is provided, which are shown below

$$\mathbf{G} = \begin{bmatrix} 1+D & 1+D^2 & 1+D+D^2 \end{bmatrix}$$

$a_1 \equiv 11|110$
$a_2 \equiv 01|011$
$a_3 \equiv 10|101$
$a_4 \equiv 00|000$

$b_1 \equiv 11|100$
$b_2 \equiv 01|001$
$b_3 \equiv 10|111$
$b_4 \equiv 00|010$

$c_1 \equiv 11|000$
$c_2 \equiv 01|101$
$c_3 \equiv 10|011$
$c_4 \equiv 00|110$

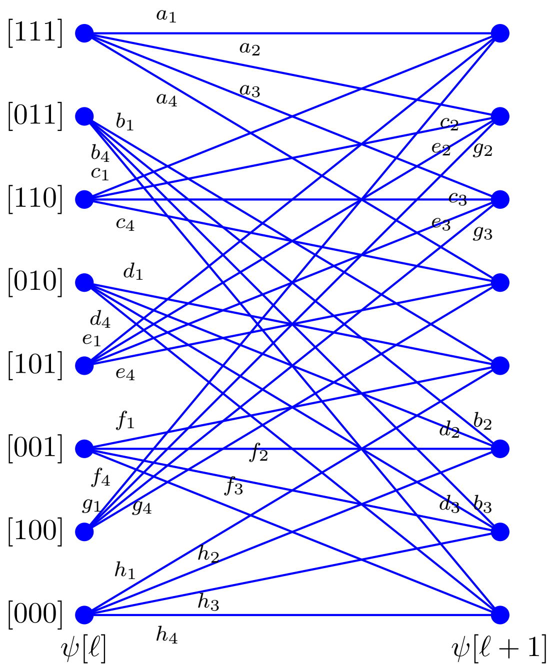
$d_1 \equiv 11|010$
$d_2 \equiv 01|111$
$d_3 \equiv 10|001$
$d_4 \equiv 00|100$

$e_1 \equiv 11|010$
$e_2 \equiv 01|111$
$e_3 \equiv 10|001$
$e_4 \equiv 00|100$

$f_1 \equiv 11|000$
$f_2 \equiv 01|101$
$f_3 \equiv 10|011$
$f_4 \equiv 00|110$

$h_1 \equiv 11|110$
$h_2 \equiv 01|011$
$h_3 \equiv 10|101$
$h_4 \equiv 00|000$

$g_1 \equiv 11|100$
$g_2 \equiv 01|001$
$g_3 \equiv 10|111$
$g_4 \equiv 00|010$



I) Obtain the schematic representation and the generator matrix for the second encoder.

II) Obtain the schematic representation and the trellis diagram for the first encoder.

III) For the first encoder, decode the bits $B^{(0)}[0]$, $B^{(0)}[1]$ and $B^{(0)}[2]$, assuming that headers with zeros have been trasmitted before and after these bits. Apply the optimal decoding algorithm if the received sequence (hard decisions) is

m	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$R[m]$	0	1	0	1	1	1	1	0	0	1	0	1	0	0	1

REMARK: clear evidence of the application of the optimal algorithm must be provided.

(2.5 points)