Exercise 1

A digital baseband communications system is designed to transmit at 6 Mbits/s using a bandwidth of 1.5 MHz. The transmitter and the receiver use normalized root raised cosine filters. A M-PAM constellation with levels $\{\pm a, \pm 3a, \ldots, \pm(M-1)a\}$ is used and the transmitted sequence $A[n]$ is white.

a) Find the order of the constellation, M, the value of a to minimize the P_e and the bandwidth of the modulated signal if the average transmitted power can not exceed 1 W and

   I) The roll-off factor is $\alpha = 0.25$.

   II) The roll-off factor is $\alpha = 0.75$.

b) Assuming a channel that only introduces amplification and delay, i.e., $h(t) = \beta\delta(t - 3T/2)$, where T is the symbol length and $|\beta| > 1$, determine if ISI exists.

c) Calculate and represent the power spectral density of the transmitted signal in the system designed in Section a.I) of this exercise.

(1.5 points)

Exercise 2

A baseband digital communications system has the following equivalent discrete channel

$$p[n] = \delta[n] + 1.5\delta[n-1] + c\delta[n-3],$$

the data sequence $A[n]$ is white and the thermal noise has a power spectral density $N_0/2$.

a) For $c = 0$ and a 2-PAM modulation with normalized levels, obtain the optimal delay of the memoryless symbol-to-symbol detector and calculate its error probability.

Now consider that $c = 0.25j$ and that the constellation is a 4-PAM with normalized levels.

b) Design a linear equalizer without length constraints using the zero forcing criterion (ZF) and obtain its error probability.

c) Now a linear equalizer with a limited number of coefficients is used.

   I) Obtain the system to be solved to design an equalizer of 4 coefficients with a delay $d = 1$ under the Minimum Mean Squared Error (MMSE).

   II) Find the approximated error probability for the best delay, indicating the value of this delay, if the equalizer coefficients are

n	0	1
$w[n]$	1	$-1/5$

REMARK: You are not required to solve the equation system, but you must provide the numerical values of all the terms involved in the solution.

(3.0 points)

Exercise 3

A digital communication system will use a 8-ary frequency modulation. The frequencies that define the pulses of the different symbols must be between 2700 MHz and 4100 MHz. Calculate the maximum bit rate (bits/s) at which it is possible to transmit, and the 8 frequencies that allow it, in the following cases:

a) A Continuous Phase Frequency Shift Keying modulation (CPFSK) is used.

b) A Minimum Shift Keying modulation (MSK) is used.

(1.5 points)

Exercise 4

A digital communication system uses an OFDM modulation with $N = 4$ subcarriers to transmit information over the band between 50 kHz and 100 kHz.

a) If the system is NOT using a cyclic prefix, what is the fastest symbol rate (bauds) that it can achieve in each subcarrier?

b) The system needs to support a VoIP call, which requires a minimum of 128 kbps overall (32 kbps in each subcarrier). Find the minimum modulation order that it will need to use, assuming that it must be a power of 2.

c) The signal is sent through a multipath wireless channel. The resulting discrete time equivalent channel experienced by the OFDM symbols is $d[m] = \delta[m] + 0.5\delta[m-2]$. What is the minimum length of the cyclic prefix that we need to add to the OFDM symbol to avoid ISI and ICI?

d) As a result of the multipath and the cyclic prefix, symbols transmitted over the first subcarrier will be received with higher power than those transmitted over the second subcarrier. Find the ratio between these two powers (remember that power is proportional to the amplitude squared).

e) Instead of using OFDM, which transmits multiple symbols in parallel over different subcarriers, we could just decrease the duration of each symbol and transmit them sequentially. What advantages does OFDM offer over the latter?

(2.0 points)

Exercise 5

Consider the two convolutional codes corresponding to the following generator matrices:

$$G_1 = \left[\begin{array}{cc} D^2+1 & D^2+D+1 \end{array} \right]$$
$$G_2 = \left[\begin{array}{ccc} 1 & D & D^2 \end{array} \right]$$

a) Are they systematic? Justify your answer.

b) Find their coding rate

c) The first code has minimum distance 5. Find the minimum distance for the second code (G_2).

d) Which of the two codes do you think is better and why?

e) Assume that your transmitter is using the second code (G_2), starting and ending at the zero state. Decode the following received word: $r = [100\ 110\ 101\ 010\ 001\ 000]$.

(2.0 points)