Exercise 1

A digital communications system will use conventional QAM constellations (crossed QAM constellations are not considered) with normalized levels to transmit at 45 Mbits/s in the frequency band between 10 and 20 MHz, where the channel behavior is ideal, and thermal noise has power spectral density $N_0/2$, with $N_0 = 10^{-21}$.

a) Design the system (carrier frequency, constellation, transmitter and receiver filters) to transmit without intersymbol inteference, using the entire frequency band with the best performance, and calculate the power of the transmitted signal.

b) Represent the power spectral density of:

   I) The modulated signal.

   II) The sampled noise at the output of the receiver filter, $z[n]$.

(2 points)

Exercise 2

A 4-PAM constellation with normalized levels is transmitted over the equivalent discrete channel

$$p[n] = \delta[n] - 5\,\delta[n-3]$$

with a sampled noise $z[n]$, which is white Gaussian and with variance $\sigma_z^2 = 0.5$, and the following observations are obtained

n	0	1	2	3	4	5	6	7	8	9
$q[n]$	$+1.2$	$+7.5$	-11.2	-5.3	$+12.7$	$+3.4$	-12.8	$+7.1$	$+0.3$	-9.4

a) For a memoryless symbol-by-symbol detector:

- I) Design the optimal detector (delay and decision regions).
- II) Calculate the conditional probability of error given that the transmitted symbol is $+1$.
- III) Obtain the decisions $\hat{A}[n]$ for $n \in \{0, 1, 2\}$.

b) For an MMSE equalizer with no restrictions on the number of coefficients:

- I) Design the equalizer, explaining how the delay is obtained.
- II) Obtain the approximated probability of error.

c) An equalizer with the following coefficients is now used

$$w[n] = -0.2\,\delta[n-1] + 0.1\,\delta[n-4]$$

- I) Obtain the optimal delay for the decision.
- II) Calculate the approximated probability of error.

(2 points)

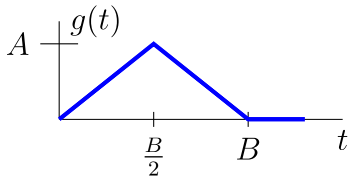
Exercise 3

a) A 4-ary frequency modulation is used to transmit at 2 Mbits/s with frequencies such that $f_i \geq 2.5$ MHz. For modulations CPFSK ("*Continuous Phase Frequency Shift Keying*") and MSK ("*Minimum Shift Keying*"):

  I) Obtain the 4 frequencies in such a way that the highest frequency is as low as possible.

  II) Obtain the effective bandwidth of the modulated signal in Hz.

b) A full response CPM modulation with modulation index 2 uses the transmitter filter $g(t)$ plotted in the figure to transmit at a binary rate of 2 Mbits/s with a 4-ary constellation ($I[n] \in \{\pm 1, \pm 3\}$).



  I) Discuss whether there is a maximum and/or minimum value for B, and calculate the value(s) of A for said value(s) of B, clearly explaining how they have been calculated.

  II) Draw the phase tree of the system for two symbol intervals, and highlight the phase evolution for $I[0] = +1$, $I[1] = -3$.

(2 point)

Exercise 4

The following sequences are known

n	0	1	2	3
$A[n]$	$+1$	-3	$+1$	-1

m	0	1	2	3
$v[m]$	$+1.1$	-0.9	-0.8	$+0.7$

a) A direct sequence spread spectrum modulation with spreading factor 4, spreading sequence

m	0	1	2	3
$x[m]$	$+1$	-1	-1	$+1$

and carrier frequency 1 MHz is employed. Transmitter filter at chip rate is a normalized root-raised cosine filter with roll-off factor 0.2.

i) Obtain the samples at chip rate associated to the transmission of data sequence $A[n]$ at a symbol rate of 5 kbauds, making explicit the discret instant associated to each sample (similarly as in the tables given above), and obtain the bandwidth of the modulated signal.

ii) Compute the observations at symbol rate $q[n]$ associated to the observations at chip rate obtained at the ouput of the receiver filter, $v[m]$, making explicit the discret instant n associated to each observation.

b) Now an OFDM modulation with 4 carriers is used to transmit data sequence $A[n]$ at a total symbol rate of 4 bauds with a carrier frequency of 1 MHz.

i) Without cyclic prefix, compute the value of the samples of the complex basebad signal at T/N associated to data sequence $A[n]$, making explicit the discret instant associated to each sample (similarly as in the tables given above), and obtain the bandwidth of the modulated signal.

ii) With a cyclic prefix of length 1 sample, compute the value of the samples of the complex baseband signal at $T/(N+C)$ associated to data sequence $A[n]$, making explicit the discret instant associated to each sample (similarly as in the tables given above), and obtain the bandwidth of the modulated signal.

(2 points)

Exercise 5

A digital communications system transmits symbols of a 2-PAM constellation with normalized levels over an ideal equivalent discrete channel and with a discrete noise with variance $\sigma_z^2 = 2$. To improve system performance, two channel codes will be used:

a) A linear block code with the following parity check matrix

$$\mathbf{H} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

i) Calculate the coding rate and calculate the number of errors that it is capable of detecting and correcting.

ii) Explain whether the code is systematic by the beginning, by the end, or by both sides.

iii) Say whether or not the code is a perfect code, clearly explaining why.

iv) Get the generator matrix of the code.

v) Obtain the table of syndromes and decode, using the syndrome based decoding and detailing each step, the received word

$$\mathbf{r} = [10101]$$

b) A convolutional code with generator matrix

$$\mathbf{G}(D) = [1 + D \quad 1 + D^2 \quad 1 + D + D^2]$$

i) Get the coding rate and the trellis diagram.

ii) Calculate the approximate probability of error working with hard output.

iii) Estimate the bits $\hat{B}_b[\ell]$ for $\ell \in \{0, 1, 2\}$, providing clear evidence of the application of the optimal algorithm, from the following received sequence (before and after the $B_b[\ell]$ bits to be estimated, the zeros corresponding to the headers have been sent).

m	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$C[m]$	1	0	1	0	1	0	1	1	1	1	0	1	0	0	1

(2 points)