DIGITAL COMMUNICATIONS
THEORY
(Time: 60 minutes. Grade 4/10)

	Grade	
Last Name(s):	1	
First (Middle) Name:	2	
ID number: .... Group	3	
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Question 1

A digital communication system uses a MSK modulation to transmit information at binary rate $R_b = 10$ kbits/s with a central or carrier frequency $\omega_c = 2\pi 10^6$ rad/s. If the system uses a quaternary modulation, with $M = 4$ symbols, determine:

a) Separation in rad/s between contiguous frequencies used by the system.

b) The value for frequencies used to transmit each symbol of the MSK modulation.

(1 point)

Question 2

A band pass modulation with symbols $A[n] \in \{\pm 1, \pm j\}$ and with transmitter filter $g(t)$ being a causal rectangular pulse of duration T seconds normalized in energy, is transmitting through a channel with complex equivalent baseband response $h_{eq}(t) = j\delta(t - T/2)$.

a) Obtain the equivalent discrete channel if the receiver uses a filter that is matched to the transmitter filter. Determine if there is ISI during transmission.

b) For the previous case, obtain the minimum distance of noiseless observations at the receiver output (received constellation).

c) Obtain the equivalent discrete channel if the receiver uses a filter that is matched to the joint response between transmitter filter and channel $(g(t) * h_{eq}(t))$. Determine if there is ISI during transmission in that case.

(1.5 points)

Question 3

A direct sequence spread spectrum modulation with spreading factor $N = 4$ uses the following spreading sequence

$$x[m] = \delta[m] + \delta[m-1] + \delta[m-2] - \delta[m-3]$$

Transmitter filter at chip rate, $g_c(t)$, is a causal squared pulse of duration equal to chip time $T_c = T/N$ seconds, and normalized. The receiver filter at chip rate is matched to $g_c(t)$, and taking into account the analog channel used for transmission, the equivalent discrete channel at chip rate $T_c = T/N$ is:

$$d[m] = \delta[m] - \frac{1}{2}\delta[m-6]$$

Initial symbols of the transmitted information sequence are

n	0	1	2	3	4	5	6	7
$A[n]$	$+1$	-1	-3	$+1$	$+3$	-3	-1	$+3$

You can consider that all previously transmitted symbols, for $n < 0$, are $A[n] = +1$.

a) Plot the modulated signal corresponding to the 4 initial symbols, $A[n]$ from $n = 0$ to $n = 3$.

b) Obtain the equivalent discrete channel at symbol rate, $p[n]$.

c) In a noiseless scenario, obtain the value for observations at the output of the receiver for the 4 initial symbols, $q[0]$, $q[1]$, $q[2]$ and $q[3]$.

(1.5 points)

DIGITAL COMMUNICATIONS
EXERCISES
(Time: 120 minutes. Grade 6/10)

Last Name(s): ....
First (Middle) Name: ....
ID Number: .... Group ....
Signature

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1	
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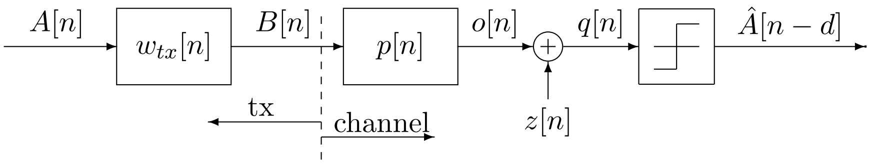
Exercise 1

A digital communication system transmits using a 2-PAM modulation, $A[n] \in \{\pm 1\}$, through equivalent discrete channel $p[n] = 0.1\ \delta[n] + 0.9\ \delta[n-1]$. Traditional systems pretending to cancel ISI use a channel equalizer $w[n]$ at the receiver. Another option is to try to cancel ISI working at the transmitter side, modifying the constellation $A[n]$ before transmitting through the channel with a filter. The idea is that when the modified constellation $B[n]$ is transmitted through channel $p[n]$, ISI is minimized according to a ZF criterion. The equalizer is moved from receiver to transmitter, as shown in the figure. In this case, a filter with two coefficients is used, $w_{tx}[n] = a\ \delta[n] + b\ \delta[n-1]$.



a) Obtain the modified constellation for $B[n]$ as a function of the coefficients of $w_{tx}[n]$.

b) Obtain the filter $w_{tx}[n]$ given by the ZF criterion for a delay $d = 1$ (remember that ZF tries to force $o[n] \approx A[n-d]$, i.e. $c[n] = w_{tx}[n] * p[n] \approx \delta[n-d]$).

c) Compute the exact probability of error when the receiver uses a memoryless symbol-by-symbol detector with the optimal delay d given by joint response $c[n]$. If response $c[n]$ has not been obtained in previous section, assume $c[n] = 0.1\ \delta[n] + \delta[n-1] + 0.01\ \delta[n-2]$ and obtain optimal value for d. Variance for noise sequence $z[n]$ is σ^2.

d) Under the same noise scenario than in previous section, compute the exact probability of error if filter $w_{tx}[n]$ is removed. Discuss if filter $w_{tx}[n]$ provides some benefit to this system.

Note:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

(3 points)

Exercise 2

This exercise will consider two channel codes working in the hard output of a digital communication system with a binary error rate in this hard output $BER = 10^{-4}$.

a) First code is a linear block code with syndrome table

$\boldsymbol{e}$							$\boldsymbol{s}$		
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	0
0	0	0	0	0	1	0	1	0	1
0	0	0	0	1	0	0	0	1	1
0	0	0	1	0	0	0	1	1	1
0	0	1	0	0	0	0	0	0	1
0	1	0	0	0	0	0	0	1	0
1	0	0	0	0	0	0	1	0	0

I) Obtain the parity check matrix associated to this table, and the generating matrices of one systematic and other non-systematic code that work using this table. Which code (systematic or non-systematic) will have better performance?

II) For the systematic code, obtain the minimum Hamming distance, explaining clearly how it is obtained, and obtain also the probability of error in the decodification of a received word.

III) Decode for the systematic code, using the syndrome based method and explaining every step of this method, the following received word

$$\boldsymbol{r} = [\ 1 \quad 0 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0\ ]$$

b) Second code is a convolutional code with generating matrix

$$\boldsymbol{G}(D) = [1 + D,\ 1 + D + D^2]$$

I) Plot the schematic representation and the trellis diagram for this code, and obtain the performance achieved with this code.

II) Decode, using the optimal decoding algorithm for convolutional codes, the following received sequence

n	0	1	2	3	4	5	6	7	8	9
$C[n]$	0	1	1	1	1	1	0	1	0	0

Assume that only 3 information bits are transmitted between cyclic headers of zeros, and that in this case the bits $B[-1] = B[-2] = 0$ and $B[3] = B[4] = 0$ correspond to these cyclic headers. Clear evidence of the application of the optimal algorithm has to be provided.

(3 puntos)