DIGITAL COMMUNICATIONS
PART A
(Time: 60 minutes. Points 4/10)

Last Name(s):	Grades	
First (Middle) Name:	1	
ID number: .... Group	2	
Signature	3	
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Exercise 1

A direct sequence spread spectrum system with spreading factor $N = 6$ has the following spreading sequence

m	0	1	2	3	4	5
$x[m]$	$+1$	-1	$+1$	-1	$+1$	-1

a) Obtain the 6 initial values ($m \in \{0, 1, \cdots, 5\}$) of data sequence at chip rate, $s[m]$, associated to the transmission of the following symbol sequence

n	0	1	2	3	4	5
$A[n]$	-3	-1	$+1$	$+3$	$+1$	-1

b) Given that equivalent discrete channel at chip rate is

$$d[m] = \delta[m] + \frac{1}{2}\delta[m-8],$$

obtain the equivalent discrete channel at symbol rate $p[n]$.

(1.25 points)

Exercise 2

A digital communication system uses a differential phase (DPSK) modulation with a constellation of $M = 4$ symbols with normalized levels

$$\boldsymbol{a}_0 = \begin{bmatrix} +1 \\ +1 \end{bmatrix}, \ \boldsymbol{a}_1 = \begin{bmatrix} -1 \\ +1 \end{bmatrix}, \ \boldsymbol{a}_2 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}, \ \boldsymbol{a}_3 = \begin{bmatrix} +1 \\ -1 \end{bmatrix}.$$

a) Plot the block diagram for the modulator, with the binary sequence at the input and the modulated signal at the output.

b) Design the binary assignment to minimize the bit error rate (BER).

c) Given the following binary sequence

ℓ	0	1	2	3	4	5	6	7
$B_b[\ell]$	0	1	1	1	1	0	0	0

obtain the sequence of transmitted symbols $A[n]$.

d) Given the following observations at the output of the demodulator

n	0	1	2	3
$q[n]$	$1.27e^{j\pi/5}$	$0.93e^{j4\pi/9}$	$2.12e^{j\pi}$	$1.56e^{j5\pi/8}$

estimate the binary sequence using a DPSK non coherent receiver.

NOTE: When some assumption is necessary, it has to be remarked in the solution.

(1.25 points)

Exercise 3

A digital communication system transmits at a binary rate 10 kbits/s and has assigned the frequency band between 5 kHz and 10 kHz. Transmitter and receiver use normalized root raised cosine filters with roll-off factor α. Constellation is a M-QAM with normalized levels, and transmitted data sequence $A[n]$ is white.

a) Obtain the carrier frequency, the power of the modulated signal, the bandwidth of the modulated signal and the constellation order, M, in the following cases:

   I) Roll-off factor is $\alpha = 0$.

   II) Roll-off factor is $\alpha = 0.75$.

b) Assuming that the channel response in the assigned frequency band is ideal, and given that $\alpha = 0.75$, plot the power spectral density of the modulated signal, with proper labels in both axes (including all necessary numerical values).

c) Given that $\alpha = 0$, now the channel has response

$$h(t) = \text{sinc}^2\left(10^4\, t\right)$$

   Obtain the equivalent discrete channel, in the time domain or in the frequency domain, and given this equivalent discrete channel discuss about if intersymbol interference will appear during transmission.

(1.5 points)

DIGITAL COMMUNICATIONS
PART B
(Time: 120 minutes. Points 6/10)

	Grades	
Last Name(s):	4	
First (Middle) Name:	5	
ID number: .... Group	T	
Signature		

Exercise 4

A digital baseband communication system has the following equivalent discrete channel

$$p[n] = \frac{1}{2}\delta[n] + \delta[n-2].$$

Constellation is a M-PAM with normalized levels and thermal noise has a power spectral density $N_0/2$ with $N_0 = 0.1$.

a) With a 2-PAM constellation, if a memoryless symbol-by-symbol detector is used, obtain the optimal delay for decision and the exact probability of error obtained with that detector.

b) With a 4-PAM constellation, design the linear equalizer without constraints in the number of coefficients, and obtain the probability of error

   I) With the zero forcing (ZF) design criterion

   II) With the minimum meand squared error (MMSE) criterion

   Compare both equalizers and explain which one has the best behavior, properly supporting the response.

c) Usign again a 4-PAM constellation, now a linear equalizer of 5 coefficients is considered

   I) Select the delay that you consider to be the more appropriate to obtain the best performance, explaining clearly the reasons to choose that value, and provide the equation system that has to be solved to obtain the coefficients of the minimim mean squared error (MMSE) equalizer (it is not necessary to solve the system, but all numerical values involved in the system to be solved have to be provided).

   II) The coefficients of the equalizer are now

n	0	1	2	3	4
$w[n]$	0	0.8	0	0.1	0

   Obtain the approximated probability of error for the optimal delay and provide the value of such delay.

(3 points)

Exercise 5

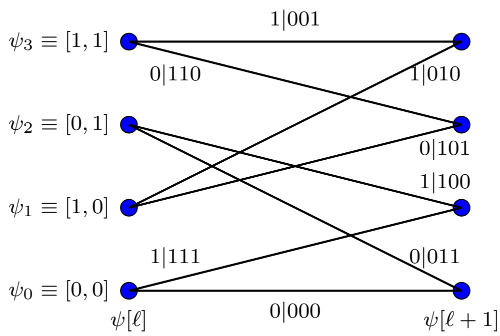
a) A linear block code has the following generating matrix

$$\mathbf{G} = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}$$

- I) Obtain the following parameters for that code:
  - Coding rate.
  - Minimum Hamming distance, explaining clearly how it was obtained, and the number of errors that the code is able to correct working with hard output.
  - Discuss if the code is perfect or not, explaining clearly the reason.
- II) Obtain the parity check matrix and the syndrome table.
- III) Using the syndrome based decoding technique (detailing each step), decode the following received word $\mathbf{r} = [1010111]$

b) Two convolutional encoders are available. For the first one, its generating matrix is known, and for the second one the trellis diagram is provided, which are the ones shown below

$$\boldsymbol{G} = \begin{bmatrix} 1 + D^2 & D & 1 \\ D & 1 + D & 1 \end{bmatrix}$$



- I) For the first encoder, obtain the schematic representation and plot the trellis diagram partially, drawing only the branches going out of the states $\psi[\ell]$ all zeros and all ones, respectively, and arriving at the corresponding states $\psi[\ell+1]$.
- II) For the second encoder, obtain the schematic representation and its generating matrix.
- III) For the second encoder, decode the bits $B^{(0)}[0]$, $B^{(0)}[1]$ and $B^{(0)}[2]$ assuming that headers of zeros are transmitted before and after these 3 bits, if the sequence of received bits is

m	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$R[m]$	0	1	0	1	1	1	1	0	0	1	0	1	0	0	1

<u>REMARK</u>: clear evidence of the application of the optimal algorithm must be provided

(3 puntos)