DIGITAL COMMUNICATIONS
PART A
(Time: 60 minutes. Points 4/10)

Last Name(s):	Grades	
First (Middle) Name:	1	
ID number: .... Group	2	
Signature	3	
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Exercise 1

A digital communication system uses an OFDM modulation with $N = 4$ carriers, with 16-QAM constellations with normalized levels in all carriers, to transmit at a total binary rate (taking into account the contribution of the 4 carriers) $R_b = 8$ Mbits/s. Joint response between transmitter, channel and receiver, sampled at T/N in the case a cyclic prefix is not used, or at $T/(N+C)$ if a cyclic prefix of C samples is used, is

$$d[m] = \delta[m] + \frac{1}{2}\delta[m-2]$$

a) Explain how to avoid inter-symbol interference (ISI) and inter-carrier interference (ICI), and provide the equivalent discrete channels at symbol rate in that case.

b) Obtain the minimum bandwidth that is necessary to transmit at the specified binary rate in the following two scenarios:

   i) The system is designed to transmit without ISI and ICI.

   ii) The system is designed to transmit using the minimum possible bandwidth, without taking care about ISI or ICI.

(1 point)

Exercise 2

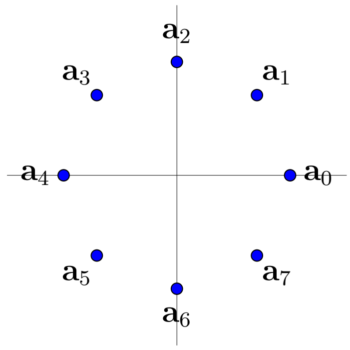
a) In a continuous phase modulation (CPM), explain the difference between a partial-response and a full-response modulation, identifying the differential feature for each variant, and provide an illustrative example for each one of them.

b) Explain how phase continuity is obtained, and write the conditions that the frequencies of the different pulses that are used have to satisfy in the following frequency modulations:

   I) Continuous phase frequency shift keying (CPFSK) modulation.

   II) Minimum shift keying (MSK) modulation.

c) A phase modulation employs a 8-PSK constellation with symbols



Provide an appropriate binary assignment in the following cases:

   I) A conventional phase shift keying (PSK) modulation is used.

   II) A differential phase shift keying (DPSK) modulation is used.

(1.5 points)

Exercise 3

Two digital communication systems will be designed with the following specifications:

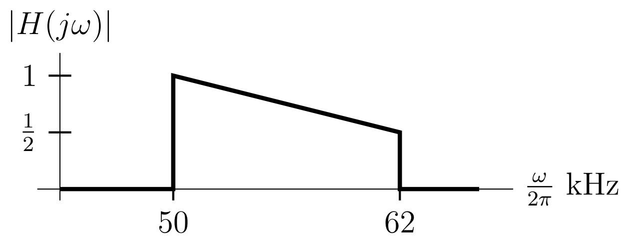
- A baseband system, using a M-PAM constellation with normalized levels, to transmit in the frequency band between zero and 12 kHz at a binary rate of 64 kbits/s.
- A band pass system, using a M-QAM constellation with normalized levels, to transmit in the frequency band between 50 kHz and 62 kHz at a binary rate of 64 kbits/s.

In both cases, transmitter and receiver will use root-raised cosine filters, and additive noise during transmission is white and Gaussian, with power spectral density $N_0/2$.

a) For the baseband system, obtain the minimum constellation order, M, and the maximum possible value for the roll-off factor at the transmitter and receiver to satisfy the specifications. Given those values, obtain the power of the modulated signal.

b) For the band pass system, obtain the carrier frequency, the minimum constellation order, M, and the maximum possible value for the roll-off factor to satisty the specifications, and given those values, plot the power spectral density of the modulated signal.

c) For the band pass system, if the channel response is the one plotted in the figure, demonstrate if inter-symbol interference is or is not present during transmission, and discuss if the noise sampled at the output of the demodulator, $z[n]$, is or is not white, explaining clearly the reason.



(1.5 points)

DIGITAL COMMUNICATIONS
PART B
(Time: 120 minutes. Points 6/10)

	Grades	
Last Name(s):	4	
First (Middle) Name:	5	
ID number: .... Group	T	
Signature		

Exercise 4

A baseband digital communication system has the following equivalent discrete channel

$$p[n] = 0.3\,\delta[n] - \delta[n-2]$$

and noise sampled at the output of the demodulator is white and Gaussian, with variance $\sigma_z^2 = 0.2$. A 2-PAM constellation with normalized levels is used and the sequence of observations at the output of the demodulator is

n	-3	-2	-1	0	1	2	3	4	5	6	7	8	9
$q[n]$	-0.6	-0.8	-0.5	$+0.2$	-0.5	$+0.3$	-0.5	$+1.1$	$+0.3$	-0.7	$+0.2$	-0.7	$+0.1$

a) If a memoriless symbol-by-symbol detector, designed to obtain the best possible performance, is used

   i) Design the optimal detector specifying all the characteristics (delay, decision regions), and obtain the decisions provided by this detector, $\hat{A}[n]$, at discrete instants $n \in \{0, 1, 2, 3, 4, 5\}$.

   ii) Obtain the exact probability of error for the system when that detector is used.

b) Now a channel equalizer, designed without constraints in the number of coefficients is considered.

   i) Obtain the channel equalizer designed with the zero forcing (ZF) criterion, and calculate the probability of error of the system ifs that receiver is used.

   ii) Explain how the optimal delay for this kind of equalizers is obtained.

c) A sequence detector is used when blocks of 3 information symbols are transmitted between cyclic headers of 2 symbols. Obtain the decoded sequence $\hat{A}[0], \hat{A}[1], \hat{A}[2]$ obtained by applying the optimal decoding algorithm, if the transmitted header implies that $A[-2] = A[-1] = A[3] = A[4] = +1$ (clear evidence of the application of the optimal decoding algorithm must be provided).

(3 points)

Exercise 5

a) A linear block code has the following generating matrix

$$\mathbf{G} = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

  I) Obtain the following parameters of the code: coding rate, minimum Hamming distance (explaining clearly how it is obtained), number of error that the code is able to detect and able to correct working with hard output.

  II) Obtain the parity check matrix and the complete syndrome table.

  III) Using syndrome based decoding, and providing evidence of each step, decode the following received word (up to provide the uncoded information bits)

$$\mathbf{r} = 1\ 1\ 1\ 1\ 1\ 0$$

b) Two convolutional codes are available. For the first one, its generating matrix is known, and for the second one its trellis diagram is provided, both shown below

$$\mathbf{G} = \begin{bmatrix} 1+D & D & 0 \\ 0 & D & 1+D \end{bmatrix}$$

$a_1 \equiv 11|001$
$a_2 \equiv 10|000$
$a_3 \equiv 01|101$
$a_4 \equiv 00|100$

$b_1 \equiv 11|010$
$b_2 \equiv 10|011$
$b_3 \equiv 01|110$
$b_4 \equiv 00|111$

$c_1 \equiv 11|111$
$c_2 \equiv 10|110$
$c_3 \equiv 01|011$
$c_4 \equiv 00|010$

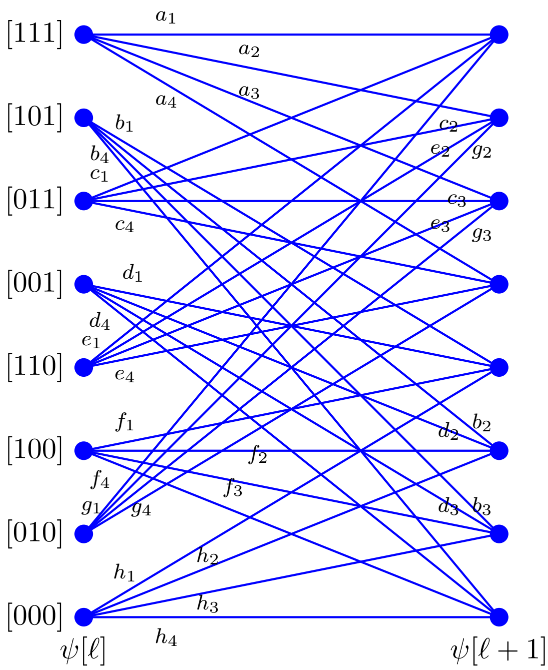
$d_1 \equiv 11|100$
$d_2 \equiv 10|101$
$d_3 \equiv 01|000$
$d_4 \equiv 00|001$

$e_1 \equiv 11|000$
$e_2 \equiv 10|001$
$e_3 \equiv 01|100$
$e_4 \equiv 00|101$

$f_1 \equiv 11|011$
$f_2 \equiv 10|010$
$f_3 \equiv 01|111$
$f_4 \equiv 00|110$

$h_1 \equiv 11|101$
$h_2 \equiv 10|100$
$h_3 \equiv 01|001$
$h_4 \equiv 00|000$

$g_1 \equiv 11|110$
$g_2 \equiv 10|111$
$g_3 \equiv 01|010$
$g_4 \equiv 00|011$



  I) Obtain the schematic representation and the trellis diagram for the first encoded (in the trellis, all branches must be drawn, but it is only necessary to include the label for branches going out of the all zeros and all ones states).

  II) Obtain the schematic representation and the generating matrix for the second encoder.

  III) For the second encoder, and assuming that previously the header with zeros that is necessary to reset the encoder has been trasmitted, obtain the encoded sequence associated to the following sequence of uncoded bits

$$B[m] = 1011001001\cdots$$

(3 points)