DIGITAL COMMUNICATIONS
PART A
(Time: 60 minutes. Points 4/10)

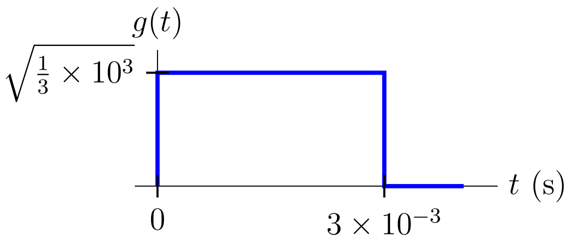
Last Name(s): ....
First (Middle) Name: ....
ID number: .... Group ....
Signature

Grades

1	
2	
3	
T	

Exercise 1

a) A baseband digital communication system has the transmitter filter $g(t)$ that is shown in the figure, and a matched filter at the receiver.

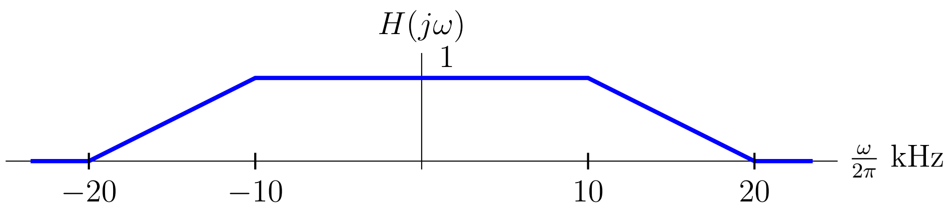


I) Obtain the maximum symbol rate without intersymbol interference if the channel is ideal.

II) Obtain the maximum symbol rate without intersymbol interference if the channel has the following impulse response

$$h(t) = \delta(t) + \delta\left(t - 4 \times 10^{-3}\right) + \delta\left(t - 10^{-2}\right).$$

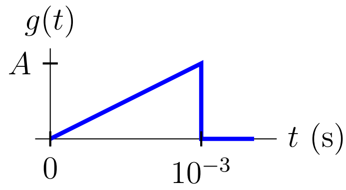
b) Now the transmitter filter is a root raised cosine, again with a matched filter at the receiver. Obtain the maximum symbol rate without intersymbol interference if the channel has the frequency response that is shown in the figure, making explicit the necessary values for the parameters of the filter (T and α).



(1 point)

Exercise 2

a) A 4-ary, $I[n] \in \{\pm 1, \pm 3\}$, continuous phase modulation, or CPM, with modulation index $h = 2$ uses the normalized transmitter filter that is shown below to transmit at a binary rate of 2 kbits/s.

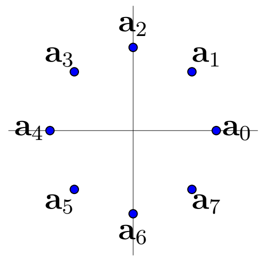


I) Explain the difference among a partial-response CPM and a full-response CPM, and identify the class of CPM modulation for this system.

II) Obtain the value of amplitude constant A.

III) Plot the phase tree for two symbol intervals.

b) A phase modulation employs a 8-PSK constellation with symbols

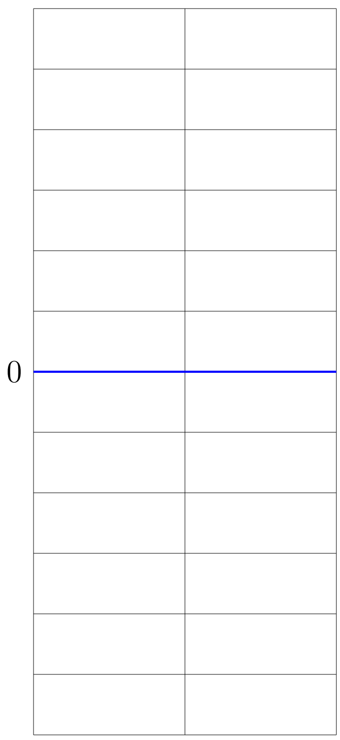


Provide an appropriate binary assignment in the following cases:

I) A conventional phase shift keying (PSK) modulation is used.

II) A differential phase shift keying (DPSK) modulation is used.

(1.5 points)



Exercise 3

A communication system uses a direct sequence spread spectrum modulation with spreading factor $N = 4$ and the spreading sequence $x[m]$ given below to transmit at a binary rate of 20 kbits/s.

m	0	1	2	3
$x[m]$	-1	$+1$	$+1$	-1

n	0	1	2	3
$A[n]$	$+1$	$+3$	-1	$+1$

a) Plot the baseband modulated signal produced to transmit the above data sequence $A[n]$ with a transmitter filter at chip rate that is a rectangular pulse, normalized, causal, and with duration equal to the chip time.

b) For the same system of the previous section, if the data is transmitted without noise through the following equivalent discrete channel at chip rate

$$d[m] = \delta[m] - \frac{1}{2}\,\delta[m-3]$$

obtain the observations at symbol rate, $q[n]$, for discrete instants $n = 1$ and $n = 2$.

c) Now the transmitter filter at chip rate is a normalized root raised cosine pulse, with roll-off factor $\alpha = 0.25$. Obtain the bandwidth of the modulated signal in the following two cases:

   I) A baseband system with a 4-PAM constellation with normalized levels.

   II) A bandpass system with a 16-QAM constellation with normalized levels and a carrier frequency $f_c = 5$ MHz.

(1.5 points)

DIGITAL COMMUNICATIONS
PART B
(Time: 120 minutes. Points 6/10)

Last Name(s):	Grades
First (Middle) Name:	4
ID number: .... Group	5
Signature	T

Exercise 4

A baseband digital communication system transmits a 2-PAM constellation with normalized levels, $A[n] \in \{\pm 1\}$. The equivalent discrete channel is

$$p[n] = \frac{1}{2}\delta[n] + \delta[n-1] + \frac{1}{2}\delta[n-2].$$

Noise added to the received signal is white and Gaussian, with power spectral density $N_0/2 = 0.01$, and the receiver filter is normalized and has an ambiguity function that satisfies the Nyquist criterion at symbol rate.

a) Obtain the optimal delay and compute the exact probability of error for a memoryless symbol-by-symbol detector.

b) In this case a channel equalizer is used at the receiver.

  I) Design the linear equalizer of 3 coefficients with the MMSE criterion for a delay $d = 2$.
  NOTE: The equation system to be solved has to be provided, with clear definition of the numerical values for of all the involved terms, but it is not necessary to solve the provided system to obtain the numerical values of the coefficients.

  II) If the coefficients of the equalizar are

$$w[0] = -0.4,\ w[1] = +1.2,\ w[2] = -0.4,$$

  obtain the approximated probability of error of this receiver.

c) Now a maximum likelihood sequence detector is used. Assuming that between each block of L data symbols a header of two known symbols is transmitted, in this case $[+1, +1]$, decode, using the optimal decoding algorithm, the data sequence of length $L = 3$, $\{A[0], A[1], A[2]\}$, if the observations at the output of the demodulator are

n	0	1	2	3	4
$q[n]$	1.6	0.2	0	-0.1	1.3

NOTE: clear evidence of the application of the decoding algorithm must be provided.

(3 points)

Exercise 5

a) A linear block code has the following parity check matrix

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & a & 1 \\ 0 & 1 & 0 & 1 & a \\ 0 & 0 & 1 & b & b \end{bmatrix}$$

I) Obtain the value of bits a and b providing the code with the best possible performance, and for these values, obtain the number of errors that the code is able to detect and to correct.

II) Obtain a compatible generating matrix in two cases:

- For a systematic code
- For a non-systematic code

III) Obtain the full syndrome table and, using the syndrome based decoding method, decode the following received word for both the systematic and non-systematic codes (every step of the decoding method has to be enumerated)

$$\mathbf{r} = 1\;1\;1\;1\;1$$

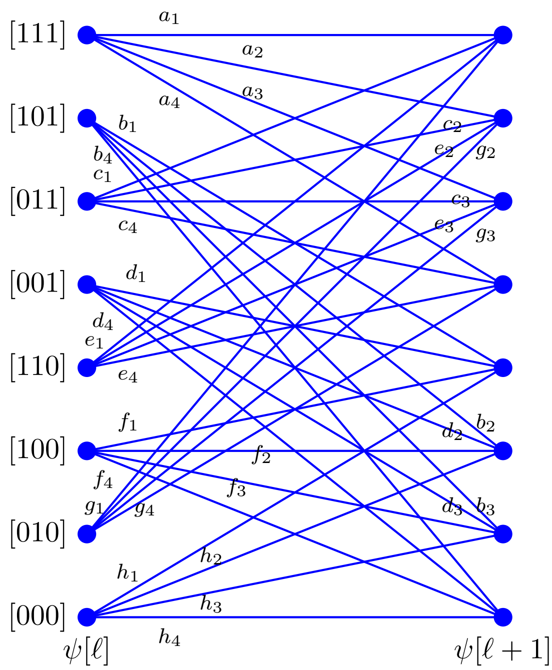
b) Two convolutional codes are available. For the first one, its generating matrix is known, and for the second one its trellis diagram is provided, both shown below

$$\mathbf{G} = \begin{bmatrix} 1+D & D & D \\ D & D & 1+D \end{bmatrix}$$

$$\begin{array}{ll} a_1 \equiv 11|001 & b_1 \equiv 11|010 \\ a_2 \equiv 10|000 & b_2 \equiv 10|011 \\ a_3 \equiv 01|101 & b_3 \equiv 01|110 \\ a_4 \equiv 00|100 & b_4 \equiv 00|111 \end{array}$$

$$\begin{array}{l} c_1 \equiv 11|111 \\ c_2 \equiv 10|110 \\ c_3 \equiv 01|011 \\ c_4 \equiv 00|010 \\ d_1 \equiv 11|100 \\ d_2 \equiv 10|101 \\ d_3 \equiv 01|000 \\ d_4 \equiv 00|001 \\ e_1 \equiv 11|000 \\ e_2 \equiv 10|001 \\ e_3 \equiv 01|100 \\ e_4 \equiv 00|101 \\ f_1 \equiv 11|011 \\ f_2 \equiv 10|010 \\ f_3 \equiv 01|111 \\ f_4 \equiv 00|110 \end{array}$$

$$\begin{array}{ll} h_1 \equiv 11|101 & g_1 \equiv 11|110 \\ h_2 \equiv 10|100 & g_2 \equiv 10|111 \\ h_3 \equiv 01|001 & g_3 \equiv 01|010 \\ h_4 \equiv 00|000 & g_4 \equiv 00|011 \end{array}$$



I) Obtain the schematic representation and the trellis diagram for the first encoder (in the trellis, all branches must be drawn, but it is only necessary to include the label for branches going out of the all zeros and all ones states).

II) Obtain the schematic representation and the generating matrix for the second encoder.

III) For the second encoder, and assuming that the header with zeros that is necessary to reset the encoder has been previously trasmitted, obtain the encoded sequence associated to the following sequence of uncoded bits

$$B[m] = 1011001001\cdots$$

(3 points)