DIGITAL COMMUNICATIONS
PART A
(Time: 60 minutes. Points 4/10)

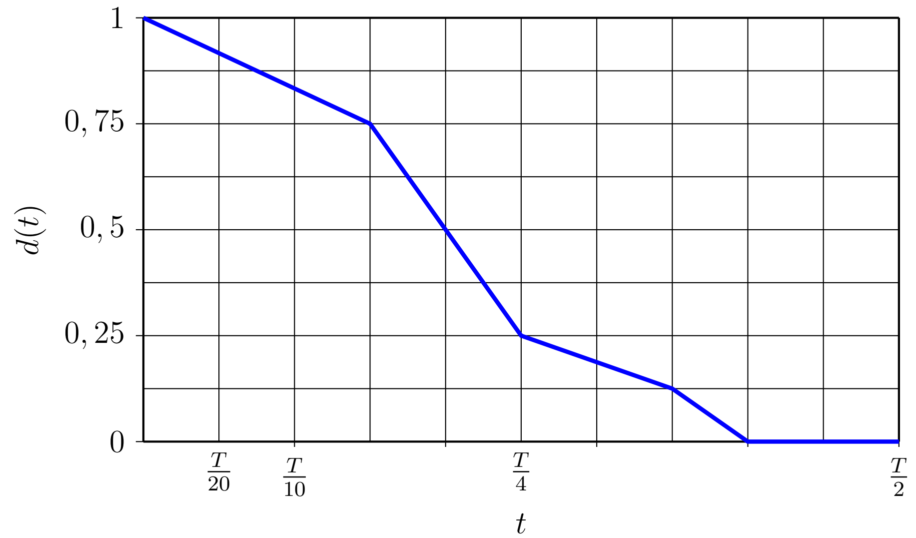
Last Name(s): ....
First (Middle) Name: ....
ID number: .... Group ....
Signature

Grades	
1	
2	
3	
T	

Exercise 1

A communication system uses a discrete time OFDM modulation with $N = 4$ carriers and with symbol length T seconds.

Joint transmitter, channel and receiver response, $d(t)$, is the one given in the picture (in a real case, this joint response depends on the reconstruction rate of the transmitter filter; for the sake of simplicity, assume that this is the response for any rate).



a) If the preliminary system does not use cyclic prefic, determine if inter-symbol interference (ISI) and/or inter-carrier interference (ICI) will be present. In the case ISI/ICI are present, design an alternative systen to avoid them being the more efficient as possible.

b) Obtain the equivalent discrete channels, $p_{k,i}[n]$, for the system designed in previous section.

(1 point)

Exercise 2

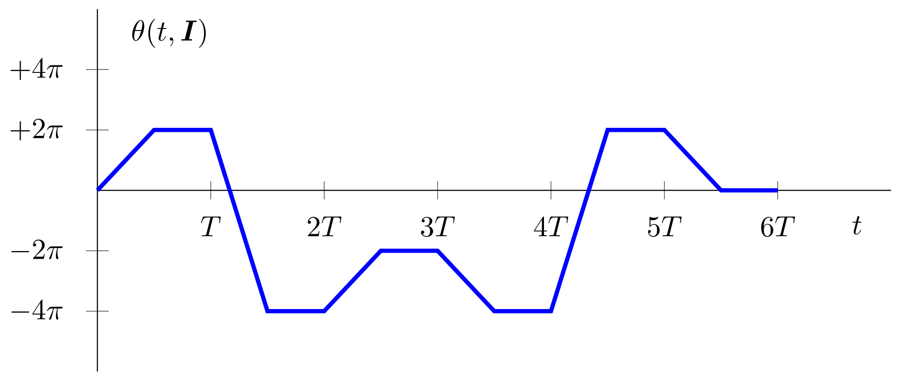
a) A communication system uses a frequency modulation with pulses using the following four frequencies

$$f_0 = 2\text{ MHz},\ f_1 = 3.5\text{ MHz},\ f_2 = 5\text{ MHz},\ f_3 = 6.5\text{ MHz}$$

i) Explain clearly the conditions that the 4 frequencies have to satisfy in a continuous phase frequency shift keying modulation (CPFSK), assess if the 4 given frequencies satisfy these conditions, and in the case of a positive answer obtain the symbol rate and binary rate of the CPFSK system.

ii) Explain clearly the conditions that the 4 frequencies have to satisfy in a minimum shift keyng modulation (MSK), assess if the 4 given frequencies satisfy these conditions, and in the case of a positive answer obtain the symbol rate and binary rate of the MSK system.

b) A continuous phase modulation (CPM) is considered now. Specifically, a quaternary full-response CPM with normalized pulse and symbols (i.e., $I[n] \in \{\pm 1, \pm 3\}$). The following picture plots the evolution of the phase of the CPM modulation, $\theta(t, \boldsymbol{I})$ during the transmission of the first six symbols.



i) Obtain the value for the modulation index, h.

ii) Obtain the sequence of the first six transmitted symbols.

iii) Obtain the normalized pulse $g(t)$.

(1.5 points)

Exercise 3

A digital communication system transmits at a binary rate of 10 kbits/s and has assigned the frequency band between 5 kHz and 10 kHz. Transmitter and receiver use normalized root raised cosine filters with roll-off factor α. The constellation is a M-QAM with normalized levels, and the transmitted data sequence $A[n]$ is white.

a) Obtain the carrier frequency, the power of the modulated signal, the bandwidth of the modulated signal and the constellation order, M, in the following cases:

   I) Roll-off factor is $\alpha = 0$.

   II) Roll-off factor is $\alpha = 0.75$.

b) Assuming that the channel response in the assigned frequency band is ideal, and given that $\alpha = 0.75$, plot the power spectral density of the modulated signal, with appropriate labels in both axes (including all necessary numerical values).

c) Given that $\alpha = 0$, now the channel has response

$$h(t) = \text{sinc}^2\left(10^4\, t\right)$$

   Obtain the equivalent discrete channel, in the time domain or in the frequency domain, and given this equivalent discrete channel discuss about if intersymbol interference happens or does not happen during transmission.

(1.5 points)

DIGITAL COMMUNICATIONS

PART B

(Time: 120 minutes. Points 6/10)

Last Name(s): ....

First (Middle) Name: ....

ID number: .... Group ....

Signature

Grades	
4	
5	
T	

Exercise 4

A digital baseband communication system has the following equivalent discrete channel

$$p[n] = \frac{1}{2}\delta[n] + \delta[n-2].$$

Constellation is a M-PAM with normalized levels and thermal noise has a power spectral density $N_0/2$ with $N_0 = 0.1$.

a) With a 2-PAM constellation, if a memoryless symbol-by-symbol detector is used, obtain the optimal delay for decision and the exact probability of error obtained with this detector.

b) With a 4-PAM constellation, design the linear equalizer without constraints in the number of coefficients, and obtain the probability of error

   I) With the zero forcing (ZF) design criterion

   II) With the minimum mean squared error (MMSE) criterion

   Compare both equalizers and explain which one has the best behavior, supporting appropriately the response.

c) Using again a 4-PAM constellation, now a linear equalizer of 5 coefficients is considered.

   I) Select the delay that is the most appropriate to obtain the best performance, explaining clearly the reasons to choose that value, and provide the equation system that has to be solved to obtain the coefficients of the minimim mean squared error (MMSE) equalizer (it is not necessary to solve the system, but all numerical values involved in the system to be solved have to be provided).

   II) The coefficients of the equalizer are now

n	0	1	2	3	4
$w[n]$	0	0.8	0	0.1	0

   Obtain the approximated probability of error for the optimal delay and provide the value of such delay.

(3 points)

Exercise 5

a) A linear block code has the dictionary given in these tables

$\mathbf{b}_i$	$\mathbf{c}_i$
0 0 0 0	0 0 0 0 0 0 0
0 0 0 1	0 1 1 1 1 0 0
0 0 1 0	1 0 1 1 0 1 0
0 0 1 1	1 1 0 0 1 1 0
0 1 0 0	1 1 1 0 0 0 0
0 1 0 1	1 0 0 1 1 0 0
0 1 1 0	0 1 0 1 0 1 0
0 1 1 1	0 0 1 0 1 1 0

$\mathbf{b}_i$	$\mathbf{c}_i$
1 0 0 0	1 1 1 1 1 1 1
1 0 0 1	1 0 0 0 0 1 1
1 0 1 0	0 1 0 0 1 0 1
1 0 1 1	0 0 1 1 0 0 1
1 1 0 0	0 0 0 1 1 1 1
1 1 0 1	0 1 1 0 0 1 1
1 1 1 0	1 0 1 0 1 0 1
1 1 1 1	1 1 0 1 0 0 1

I) Obtain the following parameters for that code:
   - Coding rate and generating matrix.
   - Minimum Hamming distance, explaning clearly how it was obtained, and the number of errors that the code is able to correct working with hard output.
   - Discuss if the code is perfect or not, explaing clearly the reason.

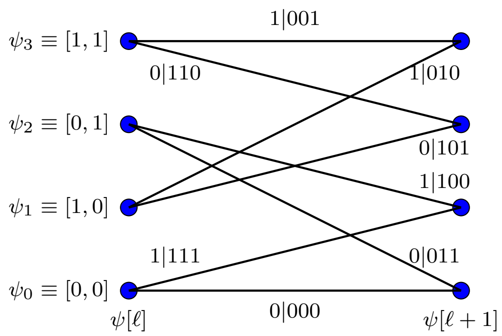
II) Obtain the parity check matrix and the syndrome table.

III) Using the syndrome based decoding technique, enumerating each step, decode the following received word

$$\mathbf{r} = 0\ 1\ 1\ 1\ 0\ 1\ 1$$

b) Two convolutional encoders are available. For the first one, its generating matrix is known, and for the second one the trellis diagram is provided, which are the ones shown below

$$\mathbf{G} = \begin{bmatrix} 1+D^2 & D & 1 \\ D & 1+D & 1 \end{bmatrix}$$



I) For the first encoder, obtain the schematic representation and plot the trellis diagram partialy, drawing only the branches going out of the states $\psi[\ell]$ all zeros and all ones, respectively, and arriving at the corresponding states $\psi[\ell+1]$.

II) For the second encoder, obtain the schematic representation and its generating matrix.

III) For the second encoder, decode the bits $B^{(0)}[0]$, $B^{(0)}[1]$ and $B^{(0)}[2]$ assuming that headers of zeros are transmitted before and after these 3 bits, if the sequence of received bits is

m	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$R[m]$	0	1	0	1	1	1	1	0	0	1	0	1	0	0	1

REMARK: clear evidence of the application of the optimal algorithm must be provided.

(3 points)