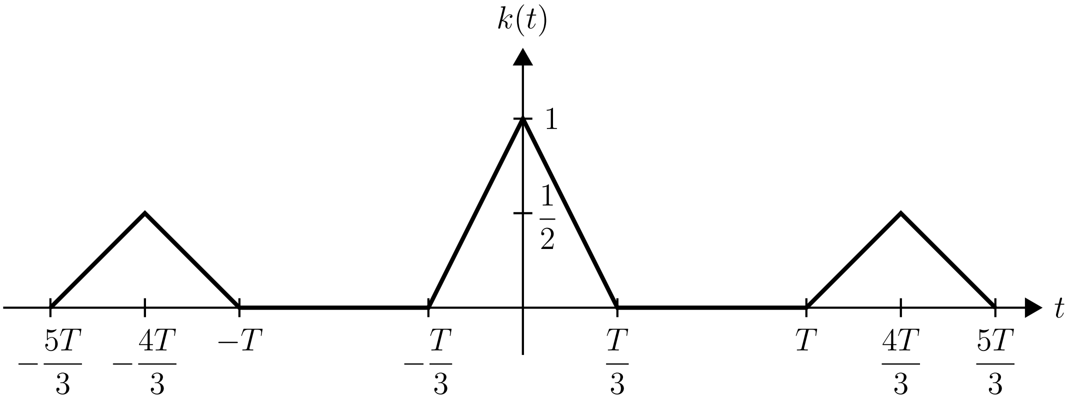
Exercise 1

A digital baseband communications system transmits a white sequence of symbols, $A[n]$, equiprobable, with a 4-PAM modulation with levels $\{\pm 1/2, \pm 3/2\}$. The joint response of the transmitter and receiver filters, $k(t)$, is shown in the figure below



where T is the symbol length, and the receiver filter is matched to the transmitter. Assuming that the channel does not distort ($h(t) = \delta(t)$) and the noise at the receiver input, $n(t)$, is white with power spectral density $N_0/2$:

a) Determine if there exists intersymbol interference (ISI)

b) Demonstrate if the sampled noise, $z[n]$, is white.

c) Obtain the power spectral density (PSD) of the transmited signal, $S_S(j\omega)$.

(1.5 points)

Exercise 2

A digital baseband communications system has the following equivalent discrete channel

$$p[n] = \delta[n] + 2\delta[n-2].$$

The transmitted constellation is a 2-PAM with normalized levels, the symbols are equiprobable and white, and the thermal noise has a power spectral density $N_0/2 = 10^{-1}$ Watts/Hz.

a) If memoryless symbol-by-symbol detector is used, obtain the optimal delay for the decision, and calculate the exact error probability that is obtained with that receiver and delay for the decision.

b) Design a linear equalizer without limitation of coefficients with the mimimum mean square error (MMSE) criterion and obtain its probability of error.

c) Now, a maximum likelihood sequence detector is used.

   I) Obtain the trellis diagram.

   II) Obtain the probability of error.

   III) Decode the maximum likelihood sequence using the optimal algorithm if the received sequence of observations is:

n	0	1	2	3	4
$q[n]$	$+2$	$+2$	0	-2	0

   when $A[n] = +1$ for $n < 0$ and for $n \geq 3$.

<u>REMARK</u>: Clear evidence of the application of the decoding algorithm must be provided.

(3.0 points)

Exercise 3

A digital communication system transmits at 2 Mbits/s using 4-ary frequency modulation.

a) If the modulation is a continuous phase frequency shift keying modulation (CPFSK)

   I) Design the system: indicate the frequencies that are used, draw the pulses for all symbols, and perform the binary assignment.

   II) Calculate the effective bandwidth of the modulation (width between main lobes).

   III) Plot the modulated signal for the transmission of the following binary sequence

m	0	1	2	3	4	5	6	7
$B_b[m]$	0	1	1	0	1	1	0	0

b) Repeat the previous section with minimum shift keying modulation (MSK).

(1.5 points)

Exercise 4

You are being asked to design a digital communication system with carrier frequency $w_c = 2\pi \times 5 \cdot 10^6$ to transmit over a multipath channel. For a signal transmitted with 1 V amplitude, this channel yields a direct line-of-sight component with amplitude 5 mV and an echo received $t_0 = 2\mu$s later with amplitude 1 mV. Your boss wants you to achieve an effective data rate above 3 Mbps with an 8-PAM modulation, but she is not willing to tolerate any inter-symbol-interference (ISI).

a) You decide to use OFDM with $N = 6$ subcarriers[1] and 8-PAM in every subcarrier. Find the equivalent discrete time channel $d[m]$ sampled at time $T/6$, where T represents the duration of an OFDM symbol.

b) Still assuming $N = 6$, you input the following symbols into the OFDM modulator:

n	0	1	2	3	4	5	6	7	8	9	10	11
A[n]	-3	+1	-3	+1	-1	-1	-3	-1	1	3	-1	1

The modulator computes the 6-IDFT, scales the output by $\sqrt{T}$, and transmits the following symbols:

n	0	1	2	3	4	5	6	7	8	9	10	11
$\sqrt{T} \cdot s[n]$	-1	$-\frac{1}{3}$	$j\frac{1}{\sqrt{3}}$	$-\frac{4}{3}$	$-j\frac{1}{\sqrt{3}}$	$-\frac{1}{3}$	0	-1	$-j\frac{1}{\sqrt{3}}$	-1	$j\frac{1}{\sqrt{3}}$	-1

You try transmitting these through the channel and observe that they suffer distortion. In order to avoid ICI and ISI you decide to use a cyclic prefix of length $C = 3$. Provide the scaled samples $\sqrt{T} \cdot s[m]$ to be transmitted in the example above.

c) How much bandwidth do you need if you are using a prefix of length $C = 3$?

(2.0 points)

[1] In practice, N is usually much larger, between 32 and 512.

Exercise 5

A certain linear block code has the following parity check matrix:

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

a) Find a generator matrix for this code. Is it systematic?

b) If you are using a BPSK modulation with $T = 10^{-6}$ seconds per symbol, what is your information rate in bits per second? Keep in mind that not all bits are information, some of them are parity.

c) Can you say something about the error correction capability t of this code? You do not need to prove an exact value. A properly justified guess will receive full credit.

d) Find a non-zero feasible codeword for this code.

e) Explain the difference between hard and soft decoding. Do both types use syndromes?

(2.0 points)