Last Name(s):	Grades	
First (Middle) Name(s):	1	
ID Number: .... Group:	2	
Signature	3	

Exercise 1

The channel of a communications system, which transmits at a binary rate of 7 Mbits/s, has the following frequency response

$$H(j\omega) = \begin{cases} 1 - \dfrac{|\omega|}{20\pi \times 10^6} & \text{si } |\omega| \leq 20\pi \times 10^6 \text{ rad/s} \\ 0 & |\omega| > 20\pi \times 10^6 \text{ rad/s} \end{cases}$$

a) A white symbol sequence of an M-PAM constellation with normalized levels is transmitted in baseband using root-raised cosine transmitter and receiver filters. Destermine the parameters of the transmitter and receiver filters and the order of the constellation that achieve the best performance using a bandwidth of 2 MHz, and calculate the power of the modulated signal.

b) For the system designed in the previous section:

   I) Obtain the equivalent discrete channel (in time or frequency) and discuss whether or not there is intersymbol interference (explain clearly why).

   II) Obtain the power spectral density of the discrete-time noise at the receiver output, $z[n]$, and discuss whether or not the noise is white (explain clearly why).

c) Now a bandpass transmission is considered, with a carrier frequency of 5 MHz, a white sequence of a 16-QAM constellation with normalized levels, and using a 2 MHz bandwidth with root-raised cosine filters at transmitter and receiver:

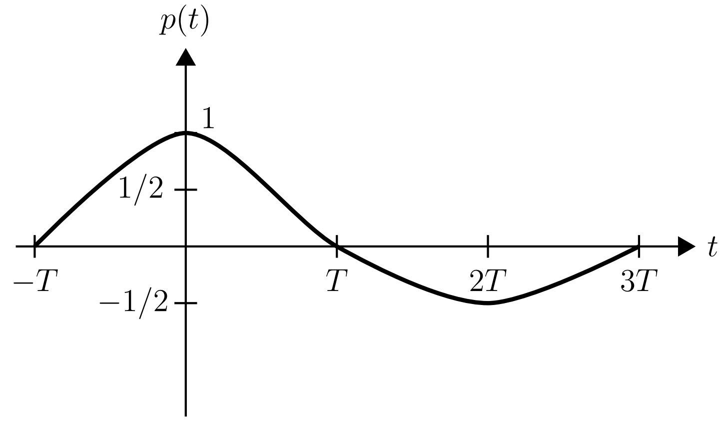
   I) Obtain the equivalent discrete channel (in time or frequency) and discuss whether or not there is intersymbol interference (explain clearly why).

   II) Obtain the power spectral density of the discrete-time noise at the receiver output, $z[n]$, and discuss whether or not the noise is white (explain clearly why).

(1.5 points)

Exercise 2

Consider a digital baseband communications system that transmits a 2-PAM modulation with levels $A[n] \in \{\pm 2\}$ at a symbol rate $R_s = 1/T$ bauds. This system has the following transmitter-channel-receiver joint response:



The variance of the discrete noise is σ_z^2.

a) Design the optimal memoryless symbol-by-symbol receiver (i.e., obtain the delay for the decision and the decision regions). Calculate the probability of error for this receiver.

b) Design the channel equalizer without coefficient limitation from the MMSE criterion and obtain the probability of error at the output of the equalizer.

c) Design the three-coefficient channel equalizer from the ZF criterion with a decision delay $d = 1$.

REMARK: You do not need to solve the system of equations, but each of the variables must be clearly defined. Similarly, if your solution involve integrals, they need not be calculated.

(2.5 points)

Exercise 3

A digital communications system uses phase modulations with the next constellation

$$\mathbf{a}_0 = \begin{bmatrix} +1 \\ +1 \end{bmatrix} \quad \mathbf{a}_1 = \begin{bmatrix} -1 \\ +1 \end{bmatrix} \quad \mathbf{a}_2 = \begin{bmatrix} -1 \\ -1 \end{bmatrix} \quad \mathbf{a}_3 = \begin{bmatrix} +1 \\ -1 \end{bmatrix}$$

a) For a PSK modulation and a DPSK modulation, determine the binary assignment and obtain the sequence of symbols for the following binary sequence

ℓ	0	1	2	3	4	5	6	7	8	9
$B_b[\ell]$	1	0	1	1	0	0	0	1	0	1

REMARK: If you must choose any parameters in the transmitter design, clearly indicate their values.

b) For a PSK modulation and for a DPSK modulation, obtain the binary sequence associated with the following sequence of observations

$$\mathbf{q}[0] = \begin{bmatrix} +0.4 \\ +0.9 \end{bmatrix} \quad \mathbf{q}[1] = \begin{bmatrix} -0.2 \\ +0.7 \end{bmatrix} \quad \mathbf{q}[2] = \begin{bmatrix} +0.9 \\ -0.8 \end{bmatrix} \quad \mathbf{q}[3] = \begin{bmatrix} -0.1 \\ -0.8 \end{bmatrix} \quad \mathbf{q}[4] = \begin{bmatrix} +0.1 \\ -0.9 \end{bmatrix}$$

REMARK: If you must choose any parameters in the receiver design, clearly indicate their values.

c) Consider now an Offset QPSK modulation, OQPSK. Explain its main characteristic (compared to the QPSK modulation), indicate how this feature is achieved, and compare the sequence of symbols to be transmitted and the decoded binary sequence with respect to the PSK and DPSK modulations of the previous sections.

d) Explain the main differences between the waveform of the modulated signal of PSK modulation and the waveform of the modulated signal of CPFSK frequency modulation.

(2 points)

Last Name(s):	Grades
First (Middle) Name(s):	4
ID Number: .... Group:	5
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Exercise 4

a) A digital communication system uses direct sequence spread spectrum with spreading factor $N = 4$ and spreading sequence

$$x[m] = \left[\begin{array}{cccc} +1 & +1 & -1 & +1 \end{array}\right]$$

to transmit information over the frequency band 830MHz – 840MHz. The transmitter filter at chip rate $g_c(t)$ is a root-raised cosine with roll-off factor $\alpha = 0.2$. The equivalent channel at chip rate is

$$d[m] = \delta[m] + \frac{1}{8}\delta[m-2].$$

I) Determine the smallest chip period T_c such that $g_c(t)$ satisfies the Nyquist criterion. What is the symbol rate of the system?

II) Suppose the information symbols to be transmitted are given by

$$A[n] = \left[\begin{array}{ccc} +3 & -1 & -3 \end{array}\right].$$

Obtain the transmitted samples at chip rate $s[m]$ for $m = 0, 1, \ldots, 11$.

III) Compute the equivalent discrete channel $p[n]$. Is there ISI? Explain why.

b) A discrete-time OFDM modulation is used with $N = 4$ subcarriers, each modulated with equiprobable QPSK symbols, and a cyclic prefix of 2 samples to transmit information over the frequency band 830MHz – 840MHz. The modulated symbols are passed through a channel with additive complex white Gaussian noise of variance N_0 and equivalent discrete channel

$$d[m] = \delta[m] + \frac{1}{8}\delta[m-2].$$

I) Explain the role of the cyclic prefix in the discrete-time OFDM modulation. What is the loss in transmission rate incurred by the cyclic prefix?

II) Obtain the equivalent discrete channels $p_{i,k}[n]$ corresponding to the 16 sub-channels. Is there ISI or ICI? Explain why.

III) What is the fastest symbol rate (bauds) that the system can achieve in each subcarrier? Determine the total bitrate the system can support.

(2 points)

Exercise 5

a) Consider the following linear block code:

$$G = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

I) Obtain the coding rate of the code.

II) How many transmission errors can the code detect? How many errors can it correct?

III) Obtain the parity-check matrix and the syndrome table.

IV) Decode the following received word using syndrome-based decoding:

$$\mathbf{r} = [\ 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1\ ].$$

b) Consider the following convolutional code:

$$G(D) = \begin{bmatrix} 1+D & D & 1 \\ 1 & 1+D & D \end{bmatrix}$$

I) Obtain the coding rate of the code.

II) Obtain the trellis diagram of the code.

III) Find the minimum Hamming distance.

IV) Assuming that the initial and final state are the zero state, i.e., $\psi_0 = [\ 0 \quad 0\ ]$, decode the message using the Viterbi algorithm when the received message is

$$\mathbf{r} = [\ 011 \quad 110 \quad 001 \quad 110\ ].$$

(2 points)