Chapter 1 : Exercises

Exercise 1.1 A baseband PAM system uses a transmitter and a receiver such that the joint response transmitter-receiver, $p(t)$, is the following triangle pulse[1]

$$p(t) = \begin{cases} 1 - \frac{|t|}{T}, & |t| \leq T \\ 0, & |t| > T \end{cases},$$

where T is the symbol period. The distortion and noise introduced by the channel can be considered negligible.

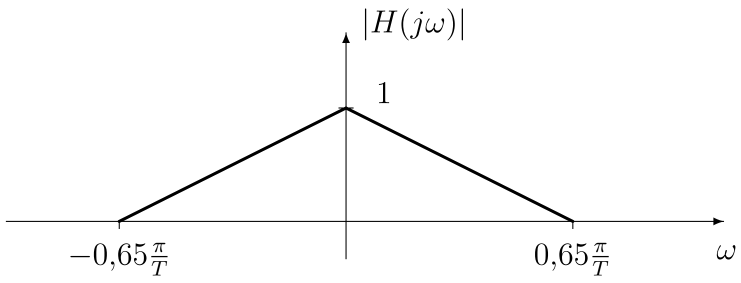
a) Demonstrate that $p(t)$ fulfills the Nyquist ISI criterion.

b) Represent the spectrum (by means of the PSD) of the signal at the output of the receiver filter when $A[n]$ is white.

c) For a 2-PAM constellation, represent the eye diagram at the output of the receiver filter.

d) For a 4-PAM constellation, represent the eye diagram at the output of the receiver filter.

Exercise 1.2 A baseband PAM communications system working at symbol time T sec. uses a root-raised cosine filter with roll-off $\alpha = 0{,}35$ as transmitter filter. The receiver uses a filter that is matched to the joint response of transmitter and channel, i.e., $f(t) = g_h(-t)$, with $g_h(t) = g(t) * h(t)$. The channel has the following frequency response [2]



If the noise is white, Gaussian, with power spectral density $N_0/2$ W/Hz, represent the power spectral density of the noise at the output of the sampler at the receiver.

Exercise 1.3 A 2-PAM constellation is transmitted through the equivalent discrete channel $p[n] = \delta[n] + 0{,}25\ \delta[n-1]$. The noise is white, Gaussian, with variance σ_z^2. Calculate the exact expression of the probability of error with a memoryless symbol-by-symbol detector.[3]

[1] Problem 5.1 of the book: A. Artés, *et al.*: Comunicaciones Digitales. Pearson Educación, 2007.
[2] Problem 5.3 of the book: A. Artés, *et al.*: Comunicaciones Digitales. Pearson Educación, 2007.
[3] Problem 5.4 of the book: A. Artés, *et al.*: Comunicaciones Digitales. Pearson Educación, 2007.

Exercise 1.4 A communications system uses a PAM modulation. The information sequence $A_w[n]$ is a white sequence. To introduce some correlation in the transmitted sequence, $A_w[n]$ is filtered with a filter with response $c[n]$, producing the sequence to be transmitted, $A[n]$, i.e., $A[n] = A_w[n] * c[n]$. The transmission filter is a root-raised cosine filter with roll-off factor $\alpha = 0$, i.e., a *sinc* function. [4]

a) Write the expression of $S_A\left(e^{j\omega}\right)$ as a function of $S_{A_w}\left(e^{j\omega}\right)$ and $C\left(e^{j\omega}\right)$.

b) Plot the power spectral density of the baseband PAM signal, $S_S(j\omega)$, in the following cases:

(I) $c[n] = \delta[n] - \delta[n-1]$

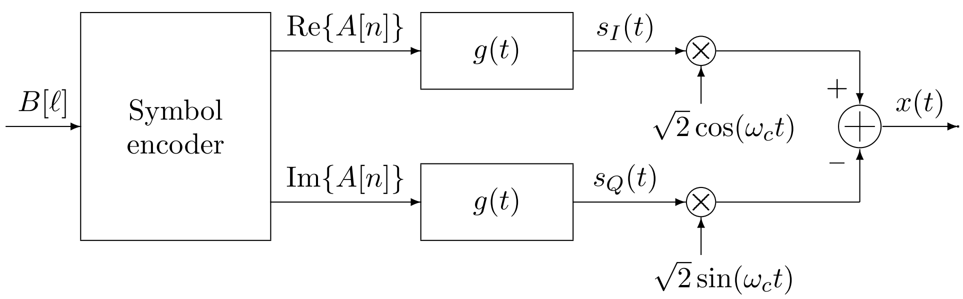
(II) $c[n] = \delta[n] + \delta[n-1]$

(III) $c[n] = \delta[n] - \delta[n-2]$

c) Obtain the transmitted power in all of these cases.

Exercise 1.5 Figure 1.1 illustrates four different cases of scattering diagram of a QPSK modulation at output of the receiver. The combined response of the filters in the transmitter and receiver is a raised-cosine filter. Explain the reason of the "dispersion" for each case in Figure 1.1.[5]

Exercise 1.6 A digital communication system uses the following modulator:



The system works in 1-4 kHz bandwidth. In this bandwidth the channel behaves like a AWGN. Design the transmitter: symbol encoder (using a QAM constellation), shaping filter at the transmitter $g(t)$ and carrier frequency ω_c to make a transmission without ISI to a binary rate of 9600 bits/s and using the whole bandwidth.

Exercise 1.7 A baseband communication system uses a BPSK constellation, $A[n] \in [\pm 1]$, and the following shaping filter

$$g(t) = \begin{cases} \frac{1}{\sqrt{T}}, & -\frac{T}{2} < t < 0 \\ -\frac{1}{\sqrt{T}}, & 0 \leq t < \frac{T}{2} \\ 0 & |t| \geq \frac{T}{2} \end{cases}$$

The modulated signal is transmitted through a linear channel with impulse response

$$h(t) = \delta(t) + \delta(t - T/2),$$

and the receiver uses a matched filter. Noise at the input of the receiver is white, Gaussian, with power spectral density $N_0/2$ W/Hz.

a) Calculate the equivalent discrete channel $p[n]$.

[4] Problem 5.6 of the book: A. Artés, *et al.*: Comunicaciones Digitales. Pearson Educación, 2007.
[5] Problem 5.12 of the book: A. Artés, *et al.*: Comunicaciones Digitales. Pearson Educación, 2007.

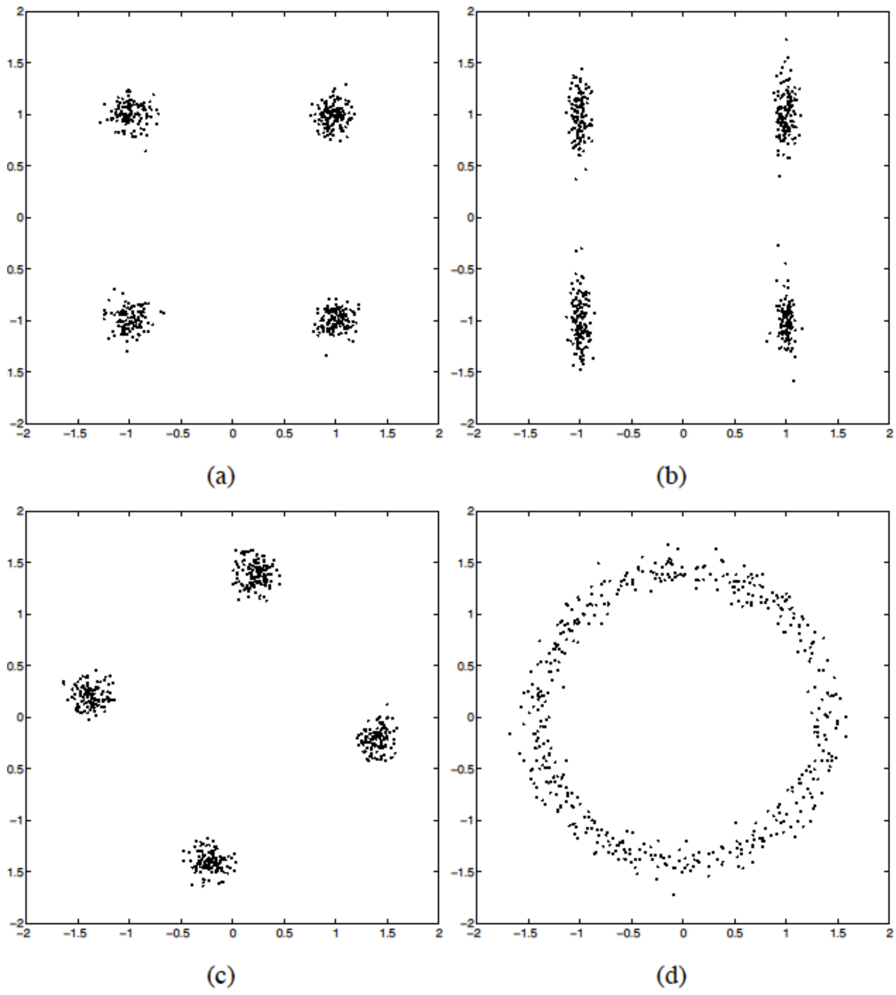
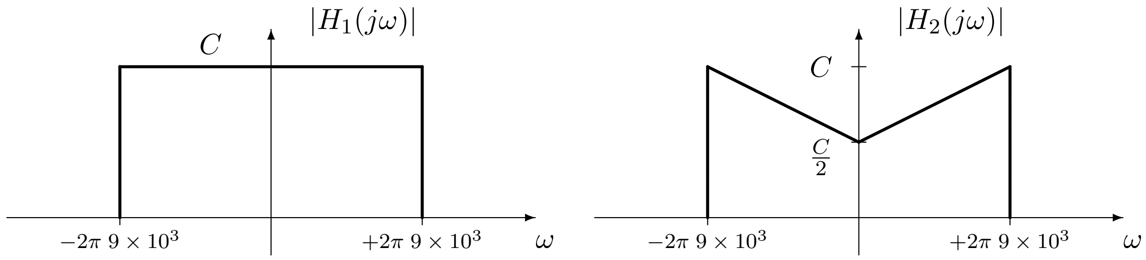


Figura 1.1: Different examples of dispersion in the scattering diagram (Exercise 1.5).

b) Obtain the power spectral density of the discrete time noise $z[n]$ present at the output of the sampler at the receiver, explaining the procedure to obtain the result.

c) Calculate the probability of error.

Exercise 1.8 A baseband transmission system sends the modulated signal through one of these channels. The receiver filter will be matched to the transmitter filter.



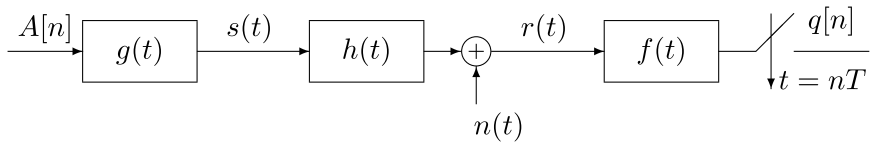
a) Design for Channel 1 the shaping filters at the transmitter and receiver in order to get no ISI and the noise at the output of the sampler be white.

b) Design for Channel 2 the shaping filters at the transmitter and receiver in order to get no ISI.

c) Design for Channel 2 the shaping filters at the transmitter and receiver in order to get white noise at the output of the sampler.

d) Get the maximum symbol rate in both channels.

Exercise 1.9 Next figure shows the block diagram for a baseband PAM system. There, $A[n]$ is the sequence of transmitted symbols, ($A[n] \in \{\pm 1\}$), $g(t)$ is a squared-root raised cosine filter, $h(t)$ is the channel impulse response, $n(t)$ is the AWGN with PSD $N_0/2$, $f(t)$ is the receiver filter, T is the symbol period and $q[n]$ are the samples at the output of the receiver.



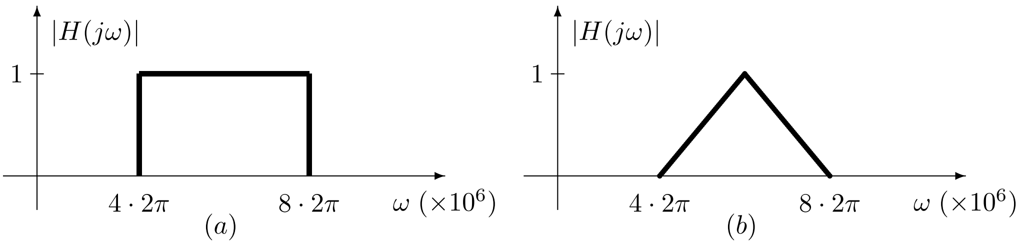
a) If $f(t)$ is designed for $k(t) = g(t) * f(t)$ to match Nyquist criteria, get $f(t)$ as a function of $g(t)$ and get the PSD for the discrete filtered noise $z[n]$.

b) If $h(t) = \delta(t) - \frac{1}{10}\delta(t-2T)$, get the impulse response of the equivalent discrete channel $p[n]$.

c) Under (b) conditions, is there ISI in the system?

d) Assuming $g(t) = f(t) = 1/\sqrt{T}$ if $|t| \leq T/2$ and $f(t) = g(t) = 0$ otherwise (that is, $f(t)$ and $g(t)$ are identical normalized squared pulses defined in $[-T/2, T/2]$), get the eye diagram in the absence of noise.

Exercise 1.10 Consider the following frequency response of two different communication channels.



a) Show if it is possible, using a QAM modulation a transmission without ISI and with white noise at the output of the sampler if we use in the receiver a matched filter to the transmitter. If your answer is positive, get the maximum transmission rate and get the shaping pulses that you would use on channel in fig. (a) and on channel in fig. (b).

b) You would like to transmit to a 10 Mbits/s bit rate using a PSK modulation over channel in fig. (a) with squared-root raised cosine filters in the transmitter and receiver. Get the minimum number of symbols needed M in the PSK modulation and the obtained symbol rate.

c) Given the constellation of previous section, obtain the feasible range of values for the roll-off factor α of the shaping filters taking into account the available bandwidth and from the range of α values get the one minimizing the effect of deviations from optimal sampling instants at the receiver.

Exercise 1.11 A communication system uses a squared-root raised cosine filter in the transmitter for a baseband PAM modulation with roll-off factor α. In the receiver there is a matched filter to the transmitter. Assume that the channel is AWGN with an impulse response $h(t)$ and noise PSD $N_0/2$. The channel bandwidth is 4 kHz.

a) Show if the sampled noise at the output of the matched filter is white.

b) Get the maximum symbol transmission rate without ISI and get the roll-off factor needed for this rate.

c) Draw the PSD of the transmitted signal in these two cases:

   I) Sequence $A[n]$ is white with mean symbol energy E_s.

   II) Sequence $A[n]$ has a PSD $S_A(e^{j\omega}) = 1 + \cos(\omega)$.

d) If the roll-off factor used is $\alpha = 0{,}25$ transmitting at the maximum symbol rate possible without ISI, get the number of symbols M needed to get a binary rate of 19200 bits per second.

Exercise 1.12 A digital communications system uses as transmitter filter $g(t)$ a root-raised cosine pulse with roll-off factor α. The receiver employs a matched filter.

a) If the transmission is performed through the linear channel with response $h(t) = \delta(t) + \frac{1}{4}\,\delta(t-2T)$, calculate the equivalent discrete channel.

b) If the channel is a baseband channel with bandwidth $B = 10$ kHz, and the desired binary rate is 54 kbits/s, using a baseband M-PAM

   (I) Calculate the minimum order of the constellation (number of symbols M) allowing to achieve the desired rate.

   (II) Calculate the symbol rate, R_s, which is necessary to obtain such binary rate with this constellation.

(III) Calculate, for this M, the value of α that allows to completely fill the available bandwidth.

c) Repeat the previous question if the channel is a bandpass channel and the modulation is a bandpass PAM using a M-QAM constellation.

Exercise 1.13 A linear baseband modulation uses a causal and normalized rectangular pulse of duration T. This modulation is transmitted through a linear channel with impulse response $h(t) = \delta(t) - 0{,}5\delta\left(t - \frac{T}{2}\right)$.

In the receiver, we consider two different scenarios. In the first case, the receiver employs a matched filter to the transmitter. In the second case, the receiver employs a matched filter to the rectangular pulse shown in Figure 1.2.

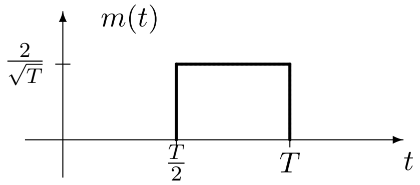


Figura 1.2: Rectangular pulse corresponding to exercise number 13.

a) Calculate the equivalent discrete channel in both cases.

b) In the second scenario, analyze if the sampled noise at the output of the (second) matched filter is white.

c) Explain, from the point of view of the ISI and sampled noise at the output of the receiver, what is the best option for the receiver.

Exercise 1.14 A digital communication system has assigned to its use the frequency range between 30 and 40 MHz. A M-QAM modulation will be used. Both transmitter and receiver will employ root-raised cosine filters with roll-off factor α.

a) Obtain the maximum symbol rate allowing to transmit without intersymbol interference (ISI), and determine the value for α that is used to achieve such maximum rate.

b) If a transmission rate of 36 Mbits/s is desired, obtain the minimum constellation order M (number of symbols in the M-QAM constellation) that is required.

c) When transmitting at the maximum symbol rate without ISI, plot the power spectral density of the transmitted signal in two cases:

   I) Sequence of data, $A[n]$, is white.

   II) Sequence of data, $A[n]$, has the following autocorrelation function

$$R_A[k] = 2\,\delta[k] + \delta[k-1] + \delta[k+1].$$

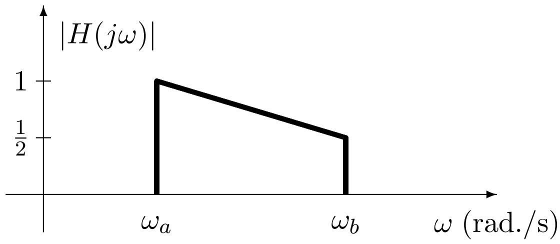
Exercise 1.15 A digital communication system has been assigned the frequency range of 10 - 15 MHz. The modulation that will be used is a 16-QAM.

a) If the transmitter uses a root-raised cosine (RRC) shaping pulse with a roll-off factor of $\alpha = 0{,}25$, the receiver is a matched filter to the transmitter and assuming that the channel frequency response is flat in the rage of frequencies used for the transmission:

I) Get the maximum symbol rate and the maximum binary rate without ISI.

II) Get the power spectral density of the modulated signal $x(t)$ if the information sequence $A[n]$ is white.

b) If in the range of frequencies assigned the channel behaves as in next figure (with $\omega_a = 2\pi \times 10 \times 10^6$ and $\omega_b = 2\pi \times 15 \times 10^6$) and the transmitter and receiver filters are as defined before:



I) Show if it is possible or not the transmission without ISI.

II) Discuss if the discrete noise at the output of the receiver $z[n]$ is white. Explain your answer.

c) For the channel of previous section and still assuming that the receiver is a matched filter to the transmitter:

I) Get the transmitter filter so that there is no ISI. The filter can be given in the time domain $g(t)$ or in the frequency domain $G(j\omega)$.

II) Discuss if in this case the discrete noise at the output of the receiver is white or not.

Exercise 1.16 A digital communication system uses a causal square pulse of length T that is normalized in energy. The receiver uses a matched filter (matched to $g(t)$). The modulated signal is transmitted through a channel whose complex equivalent baseband response is:

$$h_{eq}(t) = \delta(t) + j\delta\left(t - \frac{T}{2}\right).$$

a) Without taking into account the channel effect (i.e., $h_{eq}(t) = \delta(t)$), do the selected transmitter and receiver filters fulfill the ISI Nyquist criterion?

b) Obtain the equivalent discrete channel and the constellation at the receiver when the transmitted constellation is an orthogonal constellation with symbols $A[n] \in \{+1, +j\}$.

c) Repeat the previous section if now

$$h_{eq}(t) = j\delta(t - T).$$

Explain if in that case ISI will be present or not.

Exercise 1.17 Two digital communication systems are available. The first one is a baseband system and the second one is a bandpass system. The available range of frequencies for the first system is between 0 and 20 kHz, and the constellation is a M-PAM. The second system has been allotted the frequency range between 20 and 40 kHz, and uses a M-QAM constellation. In both systems, transmitter and receiver filters will be matched, and the transmitter filter is a root-raised cosine filter with roll-off factor α.

a) Obtain the maximum symbol rate that can be achieved in a transmission without inter-symbol interference (ISI) if the channel has an ideal behavior in its specified frequency band. Indicate the value or set of values of α that can be used to obtain such maximum rate:

   I) In the baseband system.

   II) In the bandpass system.

b) If a roll-off factor $\alpha = 0{,}25$ is used, represent the power spectral density of the transmitted signal, properly labeling each axis of the picture:

   I) In the baseband system, using a 2-PAM constellation.

   II) In the bandpass system, using a 4-QAM constellation.

c) In the bandpass system, if you pretend an ISI free transmission at binary rate of 64 kbits/s:

   I) Select the carrier frequency, ω_c, that you would use for transmission.

   II) Obtain the minimum required constellation order (number of symbols, M, in the constellation) that allows to transmit at the specified binary rate.

   III) Obtain the symbol rate used to transmit at the required binary rate when the constellation obtained in the previous section is used.

Exercise 1.18 A digital communication system has assigned for its use the band of frequencies (channel) between 800 MHz and 950 MHz. In this frequency band, the behavior of the channel is considered ideal (it does not introduce linear distortion, but only introduces white, Gaussian additive noise with power spectral density $N_0/2$). The transmitted modulation is a 16-QAM with normalized levels, and sequence $A[n]$ is white. The transmission filter is a square-root raised cosine with roll-off factor α. For the reception filter there are two possibilities:

- Filter $f_a(t)$, a square-root raised cosine with roll-off factor α.
- Filter $f_b(t)$ given by $f_b(t) = \frac{1}{\sqrt{T}}$ for $|t| \leq T/2$ and $f_b(t) = 0$ for $|t| > T/2$.

a) For $\alpha = 0{,}25$ and receiver filter $f(t) = f_a(t)$, obtain the maximum symbol rate and maximum bit rate, indicating the value for the carrier frequency ω_c that has to be used to achieve these rates transmitting in the specified frequency band.

b) For $\alpha = 0{,}25$, plot the power spectral density of the modulated transmitted signal $x(t)$. Proper labels, with numerical values, have to be included in both axes, and only the positive range of frequencies ($\omega \geq 0$ rad/s.) has to be plotted.

c) For $\alpha = 0$, demonstrate on one hand if there is or not intersymbol interference (ISI) in the transmission, and on the other hand if the sampled noise at the receiver, $z[n]$, is or not white, in the following cases:

   I) Receiver filter is $f(t) = f_a(t)$.

   II) Receiver filter is $f(t) = f_b(t)$.

Exercise 1.19 A baseband PAM modulation transmits a sequence of data $A[n] \in \{0, A\}$ with probabilities $P(A[n] = 0) = 1 - p$ and $P(A[n] = A) = p$. Autocorrelation function is given by:

$$R_A[k] = \mathbb{E}\{A[n]A[n+k]\} = A^2 p(1-p)\,\delta[k] + A^2 p^2$$

a) Calculate the mean energy per symbol of the transmitted sequence.

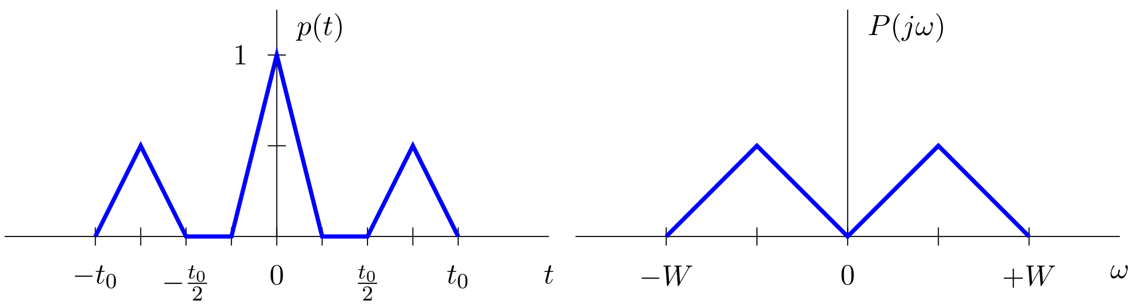
b) Obtain the power spectral density of PAM signal, and plot it with proper axis labels, if the shaping filter is a normalized root-raised cosine filter with roll-off factor $\alpha = 0{,}25$, and sequence is transmitted at symbol rate $R_s = 1$ kbauds. Provide also the bandwidth of the PAM signal.

NOTE: Some transformations

$$x[n] = \delta[n] \stackrel{TF}{\longleftrightarrow} X(e^{j\omega}) = 1$$

$$x[n] = 1,\ \forall n \stackrel{TF}{\longleftrightarrow} X(e^{j\omega}) = 2\pi \sum_k \delta(\omega + 2\pi k)$$

Exercise 1.20 A baseband communication system has its receiver filter matched to the transmission filter. Two different scenarios will be considered for transmission. The joint transmitter-channel-receiver response for these scenarios is given, respectively, in figures (a) and (b); in the first case through the joint response in the time domain, $p(t)$, and in the second case by means of the joint response in the frequency domain, $P(j\omega)$.



(a) (b)

a) Determine for each scenario if it is possible or not a transmission without intersymbol interference (ISI), and in particular

- Explain clearly the criterion that was used to determine if transmission without ISI is possible.
- In the case of a possitive response, obtain the maximum symbol rate for transmission without ISI (as a function of parameters y_0 or W in each case).

b) Consider now scenario (a), where the channel has an ideal response $h(t) = \delta(t)$.

   I) Obtain the analytic expression of shaping filter $g(t)$ in this case.

   II) Explain the condition that has to be satisfied to have the sampled noise at the output of the matched filter, $z[n]$, being white in a general case, and demonstrate if this condition is satisfied or not in this case.

Exercise 1.21 A digital communication system transmits at a binary rate 10 kbits/s and has assigned the frequency band between 5 kHz and 10 kHz. Transmitter and receiver use normalized root raised cosine filters with roll-off factor α. Constellation is a M-QAM with normalized levels, and transmitted data sequence $A[n]$ is white.

a) Obtain the carrier frequency, the power of the modulated signal, the bandwidth of the modulated signal and the constellation order, M, in the following cases:

i) Roll-off factor is $\alpha = 0$.

ii) Roll-off factor is $\alpha = 0{,}75$.

b) Assuming that the channel response in the assigned frequency band is ideal, and given that $\alpha = 0{,}75$, plot the power spectral density of the modulated signal, with proper labels in both axes (including all necessary numerical values).

c) Given that $\alpha = 0$, now the channel has response

$$h(t) = \text{sinc}^2\left(10^4\, t\right)$$

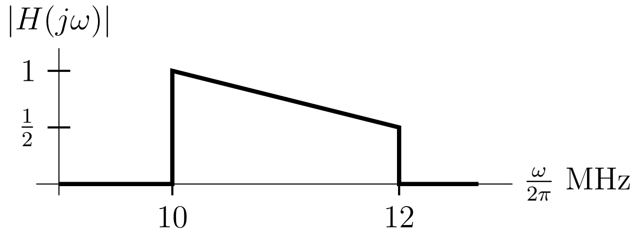
Obtain the equivalent discrete channel, in the time domain or in the frequency domain, and given this equivalent discrete channel discuss about if intersymbol interference will appear during transmission.

Exercise 1.22 A digital communication system has assigned the frequency band between 10 MHz and 12 MHz, uses matched filters in transmitter and receiver, and a 16-QAM constellation with normalized levels. Additive noise in the channel is white and Gaussian, with power spectral density $N_0/2$.

a) If the channel response in the assigned band is ideal, design the normalized transmitter filter and the carrier frequency to obtain the maximum possible binary transmission rate without intersymbol interference (ISI), and using this filter obtain the maximum binary rate, and discuss if the sampled noise at the output of the receiver, $z[n]$, is or not white explaining clearly the reason.

b) Design now the normalized transmitter filter and the carrier frequency to transmit without ISI at a binary rate of 5 Mbits/s, using the whole available bandwidth, and assuming again an ideal channel response.

c) If the channel response is the one shown in the figure, design the transmitter filter to transmit without ISI, and demonstrate if the sampled noise $z[n]$ is or not white in this case.



Exercise 1.23 Two digital communication systems will be designed with the following specifications:

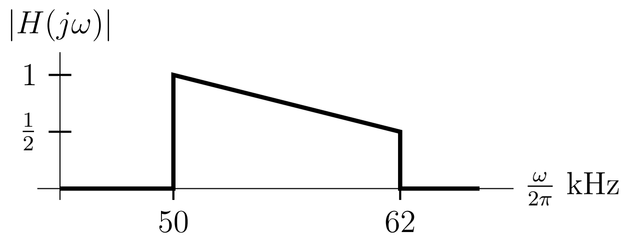
- A baseband system, using a M-PAM constellation with normalized levels, to transmit in the frequency band between zero and 12 kHz at a binary rate of 64 kbits/s.
- A band pass system, using a M-QAM constellation with normalized levels, to transmit in the frequency band between 50 kHz and 62 kHz at a binary rate of 64 kbits/s.

In both cases, transmitter and receiver will use root-raised cosine filters, and additive noise during transmission is white and Gaussian, with power spectral density $N_0/2$.

a) For the baseband system, obtain the minimum constellation order, M, and the maximum possible value for the roll-off factor at the transmitter and receiver to satisfy the specifications. Given those values, obtain the power of the modulated signal.

b) For the band pass system, obtain the carrier frequency, the minimum constellation order, M, and the maximum possible value for the roll-off factor to satisty the specifications, and given those values, plot the power spectral density of the modulated signal.

c) For the band pass system, if the channel response is the one plotted in the figure, demonstrate if inter-symbol interference is or is not present during transmission, and discuss if the noise sampled at the output of the demodulator, $z[n]$, is or is not white, explaining clearly the reason.



Exercise 1.24 A digital communication system has allotted the frequency band between 0 Hz and 5 kHz. Receiver filter is matched to the transmitter. Additive noise is white and Gaussian, with power spectral density $N_0/2$, and the transmitted data sequence is white.

a) If channel response in the allotted band is ideal, design the normalized transmitted filter to transmit at the maximum possible symbol rate without inter-symbol interference (ISI), compute the value of the maximum symbol rate, and discuss if sampled noise, $z[n]$, is or not white, explaining clearly the reason.

b) Design the normalized transmitter filter and the constellation to transmit without ISI at a binary rate of 32 kbits/s, employing the whole available bandwidth, and assuming again an ideal channel.

c) Plot the power spectral density of the modulated signal generated by the system designed in the previous section (appropriate labels, including relevant numerical values, have to be provided for both axes), and compute the power of that signal.

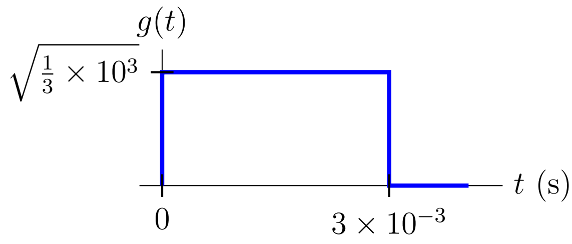
d) The channel now has the following frequency response

$$H(j\omega) = \begin{cases} 1 - \frac{|\omega|}{2\pi\times 10^4} & \text{if } |\omega| \leq 2\pi\times 10^4 \text{ rad/s} \\ 0 & \text{if } |\omega| > 2\pi\times 10^4 \text{ rad/s} \end{cases}$$

Design the transmitter filter to allow an ISI-free transmission (remember that the system can use only the frequency band between 0 and 5 kHz).

Exercise 1.25 Two different systems are analyzed:

a) A baseband digital communication system has the transmitter filter $g(t)$ that is shown in the figure, and a matched filter at the receiver.

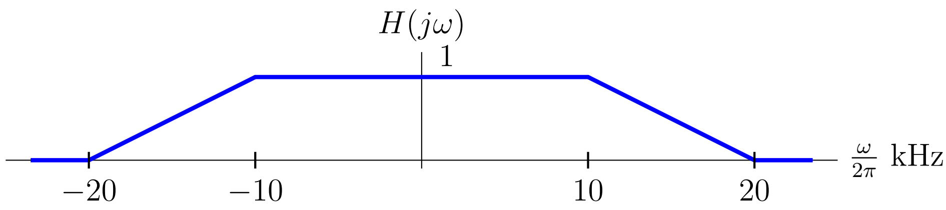


I) Obtain the maximum symbol rate without intersymbol interference if the channel is ideal.

II) Obtain the maximum symbol rate without intersymbol interference if the channel has the following impulse response

$$h(t) = \delta(t) + \delta\left(t - 4 \times 10^{-3}\right) + \delta\left(t - 10^{-2}\right).$$

b) Now the transmitter filter is a root raised cosine, again with a matched filter at the receiver. Obtain the maximum symbol rate without intersymbol interference if the channel has the frequency response that is shown in the figure, making explicit the necessary values for the parameters of the filter (T and α).



Exercise 1.26 A baseband digital communications system is designed to transmit at 6 Mbits/s using a bandwidth of 1.5 MHz. The transmitter and the receiver use normalized root raised cosine filters. A M-PAM constellation with levels $\{\pm a, \pm 3a, \ldots, \pm(M-1)a\}$ is used and the transmitted sequence $A[n]$ is white.

a) Find the constellation order, M, the value of a to minimize the P_e and the bandwidth of the modulated signal if the average transmitted power can not exceed 1 W and

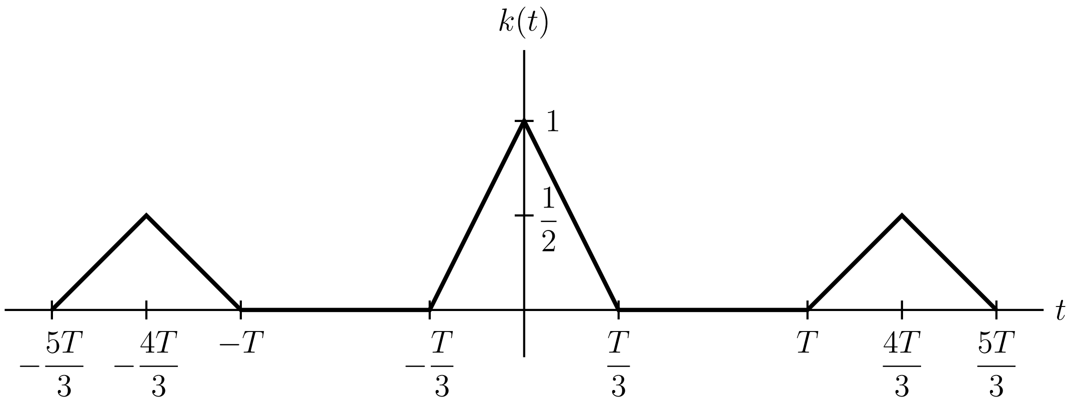
I) The roll-off factor is $\alpha = 0{,}25$.

II) The roll-off factor is $\alpha = 0{,}75$.

b) Assuming a channel that only introduces amplification and delay, i.e., $h(t) = \beta\delta(t - 3T/2)$, where T is the symbol length and $|\beta| > 1$, determine if ISI exists.

c) Calculate and plot the power spectral density of the transmitted signal in the system designed in Section a.I) of this exercise.

Exercise 1.27 A baseband digital communications system transmits a white sequence of symbols, $A[n]$, equiprobable, with a 4-PAM modulation with levels $\{\pm 1/2, \pm 3/2\}$. The joint response of the transmitter and receiver filters, $k(t)$, is shown in the figure below



where T is the symbol length, and the receiver filter is matched to the transmitter. Assuming that the channel does not distort ($h(t) = \delta(t)$) and the noise at the receiver input, $n(t)$, is white with power spectral density $N_0/2$:

a) Determine if there exists intersymbol interference (ISI).

b) Demonstrate if the sampled noise, $z[n]$, is white.

c) Obtain the power spectral density (PSD) of the transmited signal, $S_S(j\omega)$.