Chapter 2 : Exercises

Exercise 2.1 Consider the trellis diagram for a generic equivalent discrete channel with 4 non-null coefficients ($K_p = 3$), and with a 2-PAM constellation. The diagram can be divided in four disjoint sub-trellises, each one conecting two nodes at discrete instant n, and two at discrete instant $n + 1$. These sub-trellises are some times called *butterflies*[1].

a) Draw the full 8-states trellis diagram and identify the four butterflies.

Consider the general case of a M-order constellation, and a channel of length $K_p + 1$, where the trellis has M^{K_p} states.

b) Define the general butterfly by stablishing the properties of its input and output nodes.

c) How many input nodes and how many output nodes do belong to each butterfly?

d) How many butterflies are included in the trellis?

Exercise 2.2 Consider to transmit a binary symbol constellation $\{-1, +1\}$ through the channel $p[n] = 0{,}5\delta[n] - 0{,}5\delta[n-1] + 0{,}8\delta[n-2]$[2].

a) Draw the trellis diagram labeling each branch with the corresponding metric.

b) Draw the trellis diagram corresponding to the constellation of errors $\{-2, 0, +2\}$.

c) Determine the minimum distance D_{min}.

Exercise 2.3 A digital communication system has the following equivalent discrete channel:

$$p[n] = \delta[n] - \delta[n-1].$$

The power spectral density of noise in the equivalent discrete model is $\sigma_z^2 = \frac{N_o}{2} = 0{,}01$ and the transmitted modulation is a 2-PAM, $A[n] \in \{\pm 1\}$.

a) Draw the received constellation (without noise) and obtain the probability of error obtained with a symbol-by-symbol memoryless detector.

b) Draw the basic trellis diagram for the ML sequence detector.

c) Obtain the probability of error of the ML sequence detector.

[1] Problem 6.9 of the book: A. Artés, *et al.*: Comunicaciones Digitales. Pearson Educación, 2007.

[2] Problem 6.12, parts (a), (b) and (c), of the book: A. Artés, *et al.*: Comunicaciones Digitales. Pearson Educación, 2007.

d) Design the linear equalizer based on the ZF criterion and obtain the probability of error of such equalizer.

e) Design the linear equalizer based on the MMSE criterion and obtain the probability of error of such equalizer.

f) Design the linear equalizer based on the MMSE criterion with 3 coefficients and delay $d = 1$, and obtain the probability of error of such equalizer.

g) Compare the performances of the three different designed equalizers.

Exercise 2.4 A 4-PSK constellation, with equiprobable symbols $A[n] \in \{+1, -1, +j, -j\}$ and $E_s = 1$, is transmitted through the following equivalent discrete channel

$$p[n] = \delta[n] + j0{,}8\delta[n-1],$$

with additive Gaussian white noise. The goal is to evaluate the performance of the system with different receivers.

a) If in the receiver there is a memoryless symbol by symbol detector

   I) Obtain the received constellation without noise in the channel.

   II) Calculate the probability of error P_e.

b) If the receiver is composed by a linear equalizer using the ZF criterion (without constrains in its complexity) and then a symbol by symbol detector

   I) Obtain the transfer function of the equalizer.

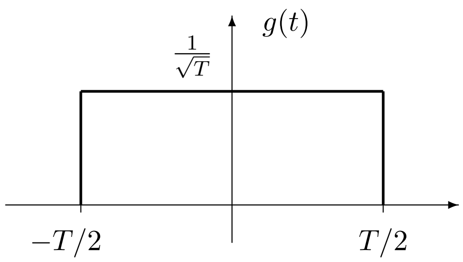
   II) Estimate P_e.

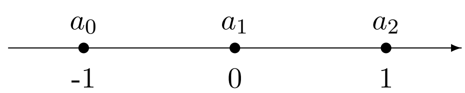
c) If the receiver is a ML (maximum likelihood) detector of sequences:

   I) Obtain the trellis diagram and the minimum distance to an erroneous event.

   II) Obtain P_e and compare with the probabilities of error previously obtained.

Exercise 2.5 A PAM based communication system PAM uses in the transmitter as shaping filter the pulse $g(t) = \frac{1}{\sqrt{T}}\Pi\left(\frac{t}{T}\right)$ shown in the figure. The system transmits the three symbols of the constellation shown in the figure with the same probability. Assume that noise is Gaussian with power spectral density $N_0/2$ Watts/Hz.





a) Obtain the equivalent discrete channel when the channel is $h(t) = \delta(t) - 0{,}5\,\delta(t - \frac{T}{2})$, and the receiver uses a filter matched to the transmitter.

b) Assume (also for the remaining parts of this problem) the equivalent discrete channel

$$p[n] = 0{,}75\ \delta[n] - 0{,}25\ \delta[n-1].$$

First of all, obtain the optimum memoryless symbol by symbol detector and calculate the probability of error.

c) Draw the trellis diagram associated to the previous equivalent discrete channel and calculate the probability of error using a ML detector of sequences. Compare its performance with the previous receiver.

d) Decode the following sequence

$$\boldsymbol{q} = [0{,}2 \quad -0{,}35 \quad -0{,}35 \quad -0{,}2].$$

using a a ML detector. Assume that $A[n] = 0$ for $n < 0$ and $n \geq 3$.

NOTE: The Fourier transform of $g(t)$ is $G(j\omega) = \sqrt{T}\text{sinc}\left(\frac{\omega T}{2\pi}\right)$.

Exercise 2.6 A communication system is described by the following equivalent discrete channel

$$p[n] = \frac{1}{2}\delta[n] - \delta[n-1] + \frac{1}{2}\delta[n-2].$$

Assume to use a 2-PAM modulation $A[n] \in [\pm 1]$, and that the noise is white, Gaussian with spectral density $N_0/2$ and $N_0 = 2\ 10^{-2}$.

a) Obtain the linear ZF (zero forcing) equalizer without constrains in the complexity and the probability of error.

b) Obtain the linear ZF (zero forcing) equalizer with 2 coefficients and delay $d = 1$.

Exercise 2.7 A communication system transmits a 2-PAM modulation, $A[n] = \pm 1$, through the following equivalent discrete channel

$$p[n] = -0{,}3\delta[n] + 0{,}8\delta[n-1] - 0{,}2\delta[n-2]$$

and discrete noise $z[n]$ white, Gaussian and with spectral density $N_0/2$.

a) Assume to use a symbol by symbol detector.

   I) Draw the constellation at the output of the channel without noise.
   II) Write the probability density function of $q[n]$ given $A[n] = 1$.
   III) Write the probability density function of $q[n]$ given $A[n] = -1$.
   IV) Assuming that the decision regions are the ones designed without ISI, obtain the probability of error P_e in two different scenarios: first assuming to take decision over $\hat{A}[n]$; second assuming to take decision over $\hat{A}[n-1]$. What is the reason of the different performances in the two scenarios?

b) Using a linear equalizer

I) Obtain the linear ZF equalizer with 3 coefficients with $d = 0$ and $d = 1$. In order to design the ZF equalizer, first obtain the channel matrix P and then the impulsive response c_d, in both cases ($d = 0$ and $d = 1$).

II) If we have a filter with coefficients $\mathbf{w}_{ZF} = [1{,}15,\ 0{,}22,\ 0{,}02]^T$ obtained for $d = 1$, find the power of the noise and of the ISI at the output of the equalizer [3].

Exercise 2.8 The equivalent discrete channel for the communication system is

$$p[n] = \frac{1}{4}\delta[n] + \delta[n-1] + \frac{1}{4}\delta[n-2].$$

The system uses a 2-PAM constellation $A[n] \in \{\pm 1\}$. The variance of the discrete noise $z[n]$ is $\sigma_z^2 = 0{,}1$.

a) If a memoryless symbol-by-symbol detector is used, choose the optimum delay d for the decision and calculate the probability of error.

b) Design a linear ZF equalizer without constrains, and calculate the corresponding probability of error.

c) Design the linear ZF and MMSE equalizers with 3 coefficients and delay $d = 2$ (write the equation system that you have to solve defining the involved terms, but it is not necessary to obtain the coefficients of the corresponding equalizer).

Exercise 2.9 To compare the performances of two different channels, the following sequence is transmitted.

n	-2	-1	0	1	2	3	4	5	6	7
$A[n]$	$+1$	$+1$	$+1$	-1	$+1$	-1	-1	-1	$+1$	$+1$

At the output of each channel without noise, we obtain the following sequences:

n	0	1	2	3	4	5	6	7
$o_1[n]$	$+0{,}1$	$-0{,}3$	$+0{,}3$	$-0{,}3$	$-0{,}1$	$-0{,}1$	$+0{,}3$	$+0{,}1$

n	0	1	2	3	4	5	6	7
$o_2[n]$	$+0{,}1$	$-0{,}3$	$+0{,}7$	$-0{,}7$	$+0{,}3$	$-0{,}1$	$+0{,}3$	$-0{,}3$

Assume also that the first channel $p[n]$ that provides $o_1[n]$ is formed by two samples (i.e., $K_{p_1} = 1$) and the second channel $p[n]$ that provides $o_2[n]$ is formed by three samples (i.e., $K_{p_2} = 2$).

a) Obtain for both channels the received constellations and determine, for each received symbols, the set of transmitted symbols (i.e., the constellation that really has been transmitted).

b) Using in both channels a memoryless detector with delay $d = 0$, determine the channel with worst performance calculating the corresponding probability of error.

c) Obtain the equivalent discrete channels $p_1[n]$ y $p_2[n]$ that have generated the received sequence.

d) Obtain the probability of error or both channels using a ML sequence detector.[4]

Without constrains for the receptor, which channel has the best performances?

[3] The symbol ^{T} denotes the transposition of a vector or matrix.

[4] For the channel $p_2[n]$ it is possible to assume that $A[n] = +1$, $\forall n$, is associated to an erroneous event with minimum Euclidean distance.

e) Obtain the sequence that has been transmitted with the highest probability if in the channel $p_2[n]$ we received:

n	0	1	2	3	4	5	6	7
$q_2[n]$	$-0{,}5$	$-0{,}2$	$-0{,}3$	$-0{,}7$	$-0{,}5$	$-0{,}2$	$+0{,}5$	$+0{,}1$

when $A[n] = +1$ for $n < 0$ and for $n > 5$.

Exercise 2.10 A baseband communication system uses as transmitter filter a causal normalized rectangular of duration T sec, and a 2-PAM constellation is transmitted through a linear Gaussian channel with spectral density $N_0/2$, with $N_0 = 0{,}02$ and the following impulse response

$$h(t) = \delta(t) - 4\,\delta\left(t - \frac{3T}{2}\right) + \frac{5}{2}\,\delta\left(t - 2T\right).$$

a) Calculate the equivalent discrete channel $p[n]$, if the receiver uses a matched filter to the transmitter filter.

b) Then, consider that the equivalent discrete channel is

$$p[n] = \delta[n] - 2\,\delta[n-1] + \frac{1}{2}\,\delta[n-2].$$

Calculate the probability of error when we use the best memoryless symbol by symbol detector.

c) Design the linear equalizer with 3 coefficients and delay in the decision $d = 1$ using ZF criterion[5].

d) Design the linear equalizer with 3 coefficients and delay $d = 3$ with the MMSE criterion.

e) If the coefficients of the equalizer are

$$w[0] = -0{,}2,\ w[1] = -0{,}6,\ w[2] = -0{,}1,$$

estimate the optimum delay for the decision and calculate the corresponding probability of error.

Exercise 2.11

When we transmit two 2-PAM (also known as BPSK) sequences, partially unknown with length $L = 4$

$$\mathbf{A}_1 = \{1, A_1[1], A_1[2], A_1[3]\} \qquad \mathbf{A}_2 = \{-1, A_2[1], A_2[2], -1\}$$

through a channel of which we know that has length 3

$$p[n] = p[0]\delta[n] + p[1]\delta[n-1] + p[2]\delta[n-2]$$

we get two different sequences in the receiver $o_1[n]$ and $o_2[n]$ when there is no noise. We also assume that $A_i[n] = +1$ for $n < 0$ and $n \geq 4$ and for $i \in \{1, 2\}$.

n	0	1	2	3	4	5
$o_1[n]$	$-0{,}1$	$-1{,}1$	$+1{,}3$	$-0{,}3$	$-0{,}1$	$-0{,}1$

n	0	1	2	3	4	5
$o_2[n]$	$-1{,}1$	$+0{,}3$	$+0{,}1$	$+0{,}1$	$+1{,}1$	$-0{,}3$

a) Get the coefficients of the channel that the two sequences went through and the trellis diagram.

b) Get the unknown values of each of the sequences.

c) If you would decide to use a memoryless detector, get the error probability. Assume that $d = 0$ in this case and that the power spectral density of the noise is given by $N_0/2$. [6]

[5] It is not necessary to solve the equation system, but you have to write the numeric values of all the terms involved in the system.

[6] If you were unable to solve a), for this item and for following solve depending on $p[0]$, $p[1]$ and $p[2]$.

d) Get the ZF equalizer with two coefficients for $d = 0$ and $d = 1$.

e) Get the sequence obtained at the output of the equalizer for $d = 1$, $u[n]$, for $n = 1$ and $n = 2$, when at the input of the equalizer we have $o_1[n]$. Did you fully recover the original sequence $\mathbf{A}_1$? Explain your answer.

Exercise 2.12 A digital communication system transmits a 2-PAM modulation, $A[n] \in \{\pm 1\}$, through the following equivalent discrete channel

$$p[n] = \frac{3}{4}\,\delta[n] + \frac{5}{4}\,\delta[n-1].$$

Discrete-time noise sampled at the demodulator output, $z[n]$, is Gaussian with variance $N_0/2$. At the receiver, two possible configurations can be used, depending on if a linear equalizer, with discrete response

$$w[n] = -\frac{12}{25}\,\delta[n] + \frac{4}{5}\,\delta[n-1]$$

is or not introduced before the detector.

a) When the equalizer is NOT used, and if detector is a memoryless symbol-by-symbol detector, select the optimal delay for decision, d, and calculate the probability of error that is obtained with this optimal delay.

b) In the structure WITH EQUALIZER, if detector is a memoryless symbol-by-symbol detector, select the optimal delay for decision, d, and calculate the probability of error that is obtained with this optimal delay (for this simple case, the exact probability of error can be obtained, without requiring the typical approximation for linear equalizers).

c) If now detector is a maximum likelihood sequence detector, used with the structure WITH EQUALIZER, i.e., applied to the equalizer output, $u[n]$, plot the trellis diagram for this detector, and obtain the approximated probability of error assuming that the sequence $A[n] = +1, \forall n$ has an erroneous event at minimum euclidean distance.

d) In the structure WITH EQUALIZER and maximum likelihood sequence detector, obtain the maximum likelihood sequence of length $L = 3$ symbols, $\mathbf{A} = [A[0], A[1], A[2]]$, by applying the optimal deconding algorith if the sequence that is received at the output of the equalizer is

$$u[0] = +1{,}11,\ u[1] = +1,\ u[2] = +1{,}26,\ u[3] = -1{,}16,\ u[4] = -1$$

by assuming that between each block of $L = 3$ symbols with information, a cyclic header of two symbols $+1$ is transmitted to cyclicly reset the state of the system.
REMARK: Clear evidence of the development of the applied algorithm has to be provided.

Exercise 2.13 Two users of a communication system wish to transmit the following sequence of equiprobable symbols $A_1[n] \in \{\pm 1\}$ and $A_2[n] \in \{\pm 2\}$, respectively. Each user transmits its sequence through a different channel ($p_1[n]$ and $p_2[n]$, respectively) and the transmitted signals are added at the receiver, where each sequence will be intended to be separated. The received observation is:

$$q[n] = A_1[n] * p_1[n] + A_2[n] * p_2[n] + z[n]$$

where $z[n]$ is white and Gaussian noise with $\sigma_z^2 = 1$, $p_1[n] = 0{,}9\delta[n] - 0{,}1\delta[n-1]$ and $p_2[n] = 0{,}8\delta[n] - 0{,}2\delta[n-1]$. If the receiver tries to recover the signal of user 1, the signal due to user 2 is an interference for user 1 (it has the same effect than intercarrier interference in an OFDM modulation). The same thing happens when the receiver tries to recover signal of user 2.

a) Obtain the noiseless received constellation and plot the trellis diagram of the whole system if you pretend to detect simultaneously both sequences (you have to plot all branches, but only have to include labels for branches starting from a single state, indicating clearly the symbols generating the transition and the output generated by the system in that transition).

b) If the receiver uses a memoryless symbol-by-symbol detector, obtain the probability of error for user 1 with delay $d = 0$, obtain the probability of error for user 2 with delay $d = 0$, compare the results explaining which user has better performance and why.

c) Design the 2 coefficients equalizer designed to recover data of user 1, unknowing the existence of user 2. Use the ZF design criterion for delay $d = 0$. Obtain the expression for the residual ISI along with the interference generated by symbols of user 2, and using them obtain an approximated expression for the probability of error for user 1.

d) Design a 2 coefficients equalizer with delay $d = 0$ using ZF criterion to recover the data of user 1, but taking now into account the existence of user 2 (extend the development of the ZF equalizer to include this information), and obtain an approximated expression for the probability of error of user 1.

Exercise 2.14 A baseband digital communication system transmits a 2-PAM constellation with normalized levels, $A[n] \in \{\pm 1\}$. Transmission if performed through a linear channel such as the equivalent discrete channel is

$$p[n] = \frac{1}{2}\delta[n] + \delta[n-1] + \frac{1}{2}\delta[n-2].$$

Noise in the received signal is white and Gaussian, with power spectral density $N_0/2 = 0{,}01$, and receiver filter is a normalized filter whose ambiguity function satisfies the conditions given by Nyquist criterion at symbol rate.

a) Obtain the optimal delay for decision with a memoriless symbol-by-symbol detector and compute the probability of error obtained with such receiver.

b) Now, a channel equalizer will be used at the receiver.

   I) Design the equalizer with 3 coefficients with the MMSE design criterion for a delay for decisions $d = 2$.

   NOTA: The equation system that has to be solved has to be stated, with the numerical values of every term included in the system, but it is not necessary to solve the system to obtain the numerical values of the equalizer.

   II) If coefficients for the equalizer are

   $$w[0] = -0{,}4,\ w[1] = +1{,}2,\ w[2] = -0{,}4,$$

   obtain the approximated probability of error.

c) A maximum likelihood sequence detector is used, assuming that between two blocks of L data symbols a header of two known symbols is transmitted, in this case $[+1, +1]$. Estimate, by using the optimal decoding algorithm, the sequence of symbols of length $L = 3$, $\{A[0], A[1], A[2]\}$, when the sequence of observations at the demodulator output is

n	0	1	2	3	4
$q[n]$	1,6	0,2	0	$-0{,}1$	1,3

NOTE: clear evidence of the application of the optimal algorithm can be provided.

Exercise 2.15 Equivalent discrete channel for a digital communication system is

$$p[n] = \frac{1}{2}\delta[n] - \frac{1}{2}\delta[n-2].$$

The system uses a 2-PAM constellation with normalized levels, $A[n] \in \{\pm 1\}$. Variance of discrete time noise $z[n]$ is $\sigma_z^2 = 0{,}1$.

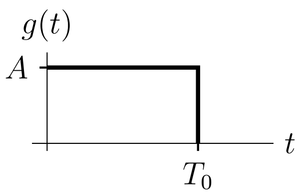
a) Using a memoriless symbol-by-symbol detector, obtain the probability of error for a delay in the decision $d = 0$ and for a delay $d = 1$.

b) Design the channel equalizer using ZF criterion without constraints in the nimber of coefficients of the equalizer, and calculate the probability of error obtained with this equalizer.

c) Design the channel equalizer using MMSE criterion without constraints in the nimber of coefficients of the equalizer, and calculate the probability of error obtained with this equalizer.

d) Design the linear equalizer of 3 coefficients using the ZF and MMSE criteria for a delay $d = 2$ (state the system of equations to be solved, including the numerical values of every involved term, but it is not necessary to solve the system to obtain the numerical values of equalizer coefficients).

Exercise 2.16 A digital baseband communication system transmits a 2-PAM constellation at a binary rate $R_b = 1$ kbits/s. The white sequence of 2-PAM equiprobable symbols, denoted as $A[n]$, is filtered to obtain sequence $B[n] = A[n] * h_c[n]$, where $h_c[n] = \delta[n] + 0{,}3\delta[n-1]$. Finally, sequence $B[n]$ is used at the input of the transmitter filter to generate the modulated baseband signal $s(t)$. Thermal noise has power spectral density $N_0/2$ with $N_0 = 0{,}1$. Transmitter filter is given in the picture



a) Assuming an ideal channel ($h(t) = \delta(t)$) and that a matched filter is used at the receiver ($f(t) = g(-t)$), obtain the value (or values) for T_0 allowing a communication free of intersymbol interference (ISI).

b) Obtain the power spectral density of $s(t)$ for $T_0 = \frac{1}{2R_b}$. Plot this power spectral density (approximately), properly labeling both axes.

c) If now the channel is not ideal and equivalent discrete channel is $p[n] = \delta[n] + 0{,}75\delta[n-1]$, obtain the optimal delay and decision regions to detect sequence $A[n]$ from observations $q[n]$ if a memoryless symbol-by-symbol detector is used. Assume that SNR is relatively high.

d) For the equivalent discrete channel of previous section, design the channel equalizer with 3 coefficients and MMSE criterion for a delay $d = 2$ designed to recover $A[n]$ from $q[n]$.

REMARK: It is not necessary to solve the system, but all numerical values involved in the system to be solved have to be provided.

e) Explain how to select the optimal delay for the MMSE equalizer (just explain the procedure, it is not necessary to calculate it).

Exercise 2.17 A digital baseband communication system has the following equivalent discrete channel

$$p[n] = \frac{1}{2}\delta[n] + \delta[n-2].$$

Constellation is a M-PAM with normalized levels and thermal noise has a power spectral density $N_0/2$ with $N_0 = 0{,}1$.

a) With a 2-PAM constellation, if a memoryless symbol-by-symbol detector is used, obtain the optimal delay for decision and the exact probability of error obtained with that detector.

b) With a 4-PAM constellation, design the linear equalizer without constraints in the number of coefficients, and obtain the probability of error

  I) With the zero forcing (ZF) design criterion

  II) With the minimum meand squared error (MMSE) criterion

  Compare both equalizers and explain which one has the best behavior, properly supporting the response.

c) Usign again a 4-PAM constellation, now a linear equalizer of 5 coefficients is considered

  I) Select the delay that you consider to be the more appropriate to obtain the best performance, explaining clearly the reasons to choose that value, and provide the equation system that has to be solved to obtain the coefficients of the minimim mean squared error (MMSE) equalizer (it is not necessary to solve the system, but all numerical values involved in the system to be solved have to be provided).

  II) The coefficients of the equalizer are now

n	0	1	2	3	4
$w[n]$	0	0,8	0	0,1	0

  Obtain the approximated probability of error for the optimal delay and provide the value of such delay.

Exercise 2.18 A baseband digital communication system has the following equivalent discrete channel

$$p[n] = \frac{1}{4}\,\delta[n] - 2\,\delta[n-1] + \frac{1}{4}\,\delta[n-2]$$

and sampled noise at the output of the demodulator is white and Gaussian with variance $\sigma_z^2 = 0{,}01$.

a) In this case a memoryless symbol-by-symbol detector, designed to obtain the best possible performance is used.

  I) If the transmitted constellation is a 4-PAM with normalized levels, design the optimal memoryless symbol-by-symbol detector providing clearly all parameters (delay, decision regions,...).

  II) If the transmitted constellation is a 2-PAM with normalized levels, obtain the exact probability of error of the system.

b) Now a channel equalizer, without constraints in the number of coefficients, is employed.

  I) Design the equalizer using the zero forcing (ZF) criterion, and obtain the probability of error if the transmitted constellation is a 4-PAM constellation with normalized levels.

II) Explain how the optimal delay is obtained for this kind of equalizers.

c) Finally, a channel equalizer with 3 coefficients is used, with coefficients

n	0	1	2
$w[n]$	$-0{,}1$	$-0{,}5$	$-0{,}1$

I) Obtain the optimal delay for the decision with this receiver, explaining clearly how it was obtained.

II) Estimate the probability of error achieved with this receiver, if the transmitted constellation is a 4-PAM with normalized levels.

Exercise 2.19 A baseband digital communication system has the following equivalent discrete channel

$$p[n] = 0{,}3\ \delta[n] - \ \delta[n-2]$$

and noise sampled at the output of the demodulator is white and Gaussian, with variance $\sigma_z^2 = 0{,}2$. A 2-PAM constellation with normalized levels is used and the sequence of observations at the output of the demodulator is

n	-3	-2	-1	0	1	2	3	4	5	6	7	8	9
$q[n]$	$-0{,}6$	$-0{,}8$	$-0{,}5$	$+0{,}2$	$-0{,}5$	$+0{,}3$	$-0{,}5$	$+1{,}1$	$+0{,}3$	$-0{,}7$	$+0{,}2$	$-0{,}7$	$+0{,}1$

a) If a memoriless symbol-by-symbol detector, designed to obtain the best possible performance, is used

I) Design the optimal detector specifying all the characteristics (delay, decision regions), and obtain the decisions provided by this detector, $\hat{A}[n]$, at discrete instants $n \in \{0, 1, 2, 3, 4, 5\}$.

II) Obtain the exact probability of error for the system when that detector is used.

b) Now a channel equalizer, designed without constraints in the number of coefficients is considered.

I) Obtain the channel equalizer designed with the zero forcing (ZF) criterion, and calculate the probability of error of the system ifs that receiver is used.

II) Explain how the optimal delay for this kind of equalizers is obtained.

c) A sequence detector is used when blocks of 3 information symbols are transmitted between cyclic headers of 2 symbols. Obtain the decoded sequence $\hat{A}[0], \hat{A}[1], \hat{A}[2]$ obtained by applying the optimal decoding algorithm, if the transmitted header implies that $A[-2] = A[-1] = A[3] = A[4] = +1$ (clear evidence of the application of the optimal decoding algorithm must be provided).

Exercise 2.20 A baseband digital communication system has the following equivalent discrete channel

$$p[n] = \delta[n] + \frac{1}{4}\ \delta[n-1] - 4\ \delta[n-2]$$

sampled noise $z[n]$ is white, Gaussian with variance $\sigma_z^2 = 0{,}2$, and M-PAM constellations with normalized levels are used.

a) In this section, a memoryless symbol-by-symbol detector is used

I) If the transmitted constellation is a 4-PAM, design the optimal detector providing all parameters (delay, decision regions,...), and compute decisions $\hat{A}[n]$ for $n \in \{0, 1, 2, 3\}$ if observations are:

n	0	1	2	3	4	5	6	7	8	9
$q[n]$	+0,7	+3,8	+1,3	−7,4	−12,1	+2,5	−15,3	+17,4	−0,1	+0,25

II) For a 2-PAM constellation, design the optimal detector providing all parameters, and compute the exact probability of error of that detector.

b) Now a channel equalizar with 3 coefficients is used and a 4-PAM is transmitted.

I) Obtain the coefficients of the equalizer using the MMSE design criterion for a delay $d = 2$.

II) Obtain the optimal delay for decision (explaining how this value is obtained) and compute the probability of error if equalizer is $w[n] = -0{,}3\,\delta[n] + 0{,}1\,\delta[n-1] - 0{,}2\,\delta[n-2]$.

c) Finally, a maximum likelihood sequence detector is used when a 2-PAM is transmitted. All symbols included in the necessary cyclic header have the value $A[n] = +1$.

I) Obtain the trellis diagram for this detector.

II) Estimate the probability of error obtained with this detector.

Exercise 2.21 A baseband digital communication system transmits a 2-PAM constellation with normalized levels, $A[n] \in \{\pm 1\}$. The equivalent discrete channel is

$$p[n] = \frac{1}{2}\delta[n] + \delta[n-1] + \frac{1}{2}\delta[n-2].$$

Noise added to the received signal is white and Gaussian, with power spectral density $N_0/2 = 0{,}01$, and the receiver filter is normalized and has an ambiguity function that satisfies the Nyquist criterion at symbol rate.

a) Obtain the optimal delay and compute the exact probability of error for a memoryless symbol-by-symbol detector.

b) In this case a channel equalizer is used at the receiver.

I) Design the linear equalizer of 3 coefficients with the MMSE criterion for a delay $d = 2$.
NOTE: The equation system to be solved has to be provided, with clear definition of the numerical values for of all the involved terms, but it is not necessary to solve the provided system to obtain the numerical values of the coefficients.

II) If the coefficients of the equalizar are

$$w[0] = -0{,}4,\ w[1] = +1{,}2,\ w[2] = -0{,}4,$$

obtain the approximated probability of error of this receiver.

c) Now a maximum likelihood sequence detector is used. Assuming that between each block of L data symbols a header of two known symbols is transmitted, in this case $[+1, +1]$, decode, using the optimal decoding algorithm, the data sequence of length $L = 3$, $\{A[0], A[1], A[2]\}$, if the observations at the output of the demodulator are

n	0	1	2	3	4
$q[n]$	1,6	0,2	0	−0,1	1,3

NOTE: clear evidence of the application of the decoding algorithm must be provided.

Exercise 2.22 A digital baseband communications system has the following equivalent discrete channel

$$p[n] = \delta[n] + 2\,\delta[n-2].$$

The transmitted constellation is a 2-PAM with normalized levels, the symbols are equiprobable and white, and the thermal noise has a power spectral density $N_0/2 = 10^{-1}$ Watts/Hz.

a) If memoryless symbol-by-symbol detector is used, obtain the optimal delay for the decision, and calculate the exact error probability that is obtained with that receiver and delay for the decision.

b) Design a linear equalizer without limitation of coefficients with the mimimum mean square error (MMSE) criterion and obtain its probability of error.

c) Now, a maximum likelihood sequence detector is used.

   I) Obtain the trellis diagram.

   II) Obtain the probability of error.

   III) Decode the maximum likelihood sequence using the optimal algorithm if the received sequence of observations is:

n	0	1	2	3	4
$q[n]$	$+2$	$+2$	0	-2	0

   when $A[n] = +1$ for $n < 0$ and for $n \geq 3$.

REMARK: Clear evidence of the application of the decoding algorithm must be provided.

SOME INTERESTING RELATIONSHIPS

Inverse of a 2×2 matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad A^{-1} = \frac{1}{D}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}, \quad \text{con } D = a\,d - b\,c$$

Some integrals, for $|a| \geq |b|$ and n integer,

$$\int_{-\pi}^{\pi} \frac{1}{a + b\ \cos(n\,\omega)} d\omega = \frac{2\pi}{\sqrt{a^2 - b^2}}$$

$$\int_{-\pi}^{\pi} \frac{1}{(a + b\ \cos(n\,\omega))^2} d\omega = \frac{2\pi a}{\sqrt{(a^2 - b^2)^3}}$$