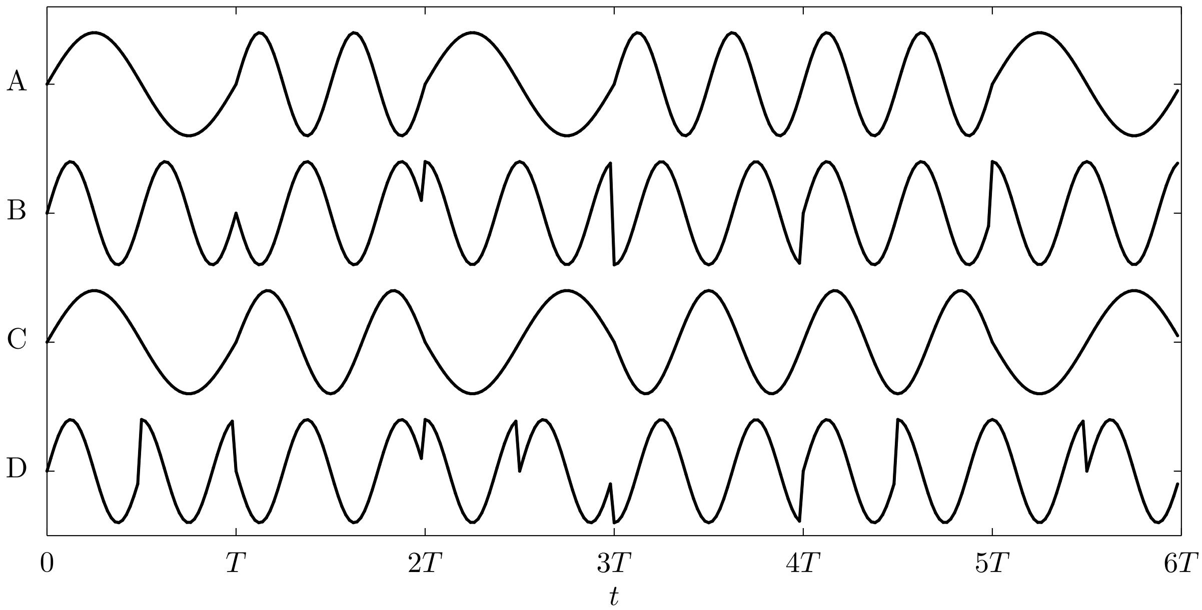
Chapter 3 : Exercises

Exercise 3.1 The following figure represents the signal corresponding to different angle modulations: QPSK, OQPSK, CPFSK y MSK.



a) For each signal, associate the corresponding modulation and explain the answer.

b) If the modulation is CPFSK or MSK, obtain de sequence of symbols $I[n] \in \{\pm 1\}$ (assume $I[0] = -1$ in both cases).

Exercise 3.2 A continuos phase modulation (CPM), with modulation index $h = 2$ and a 2-PAM constellation ($I[n] \in \{\pm 1\}$), uses the following normalized pulse

$$g(t) = \begin{cases} A, & 0 \leq t < \dfrac{T}{3} \\ A, & \dfrac{2T}{3} \leq t < T \\ 0, & \text{otherwise} \end{cases}$$

a) Find the value of A. Then, say if this continuos phase modulation (CPM) is full-response or partial-response CPM, and explain the difference.

b) Plot the phase tree for 4 symbol periods, properly labelling both axes, and also highlight (on the tree) the phase evolution corresponding to the sequence $I[0] = +1, I[1] = -1, I[2] = -1, I[3] = +1$.

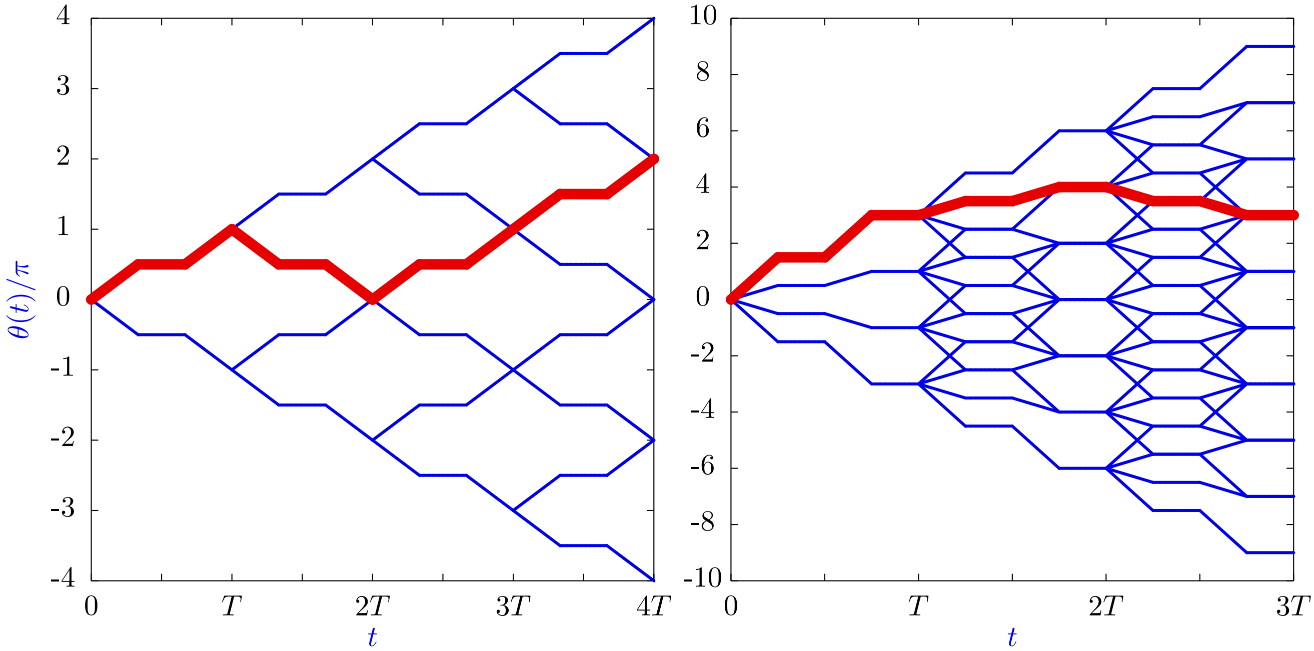


Figura 3.1: Phase trees for the CPM modulation.

Exercise 3.3 Two systems with full-response CPM modulation and modulation index $h = 1$, have the phase trees shown in Figure 3.1 (note that the phases in the Figure are scaled by the factor π in both cases).

a) For the first system

   I) Indicate the number of possible values of $I[n]$ (i.e., provide the order of the constellation M), and also provide the M values that $I[n]$ can take.

   II) Plot, properly labeling both axes, the pulse $g(t)$.

   III) Obtain the sequence of symbols $I[n]$ corresponding to the path remarked in the tree.

b) Repeat the previous steps for the second system.

Exercise 3.4 Answer the following questions related to angular modulations:

a) Which is the minimum shift among the frequencies of the shaping pulses for the following modulations?

   I) CPFSK modulation (*Continuous Phase Frequency Shift Keying*)

   II) MSK modulation (*Mimimum Shift Keying*)

b) How are the 180º phase shifts eliminated for a OQPSK modulation?

c) A CPM (*Continuous Phase Modulation*) with a modulation index $h = 2$ uses the following pulse

$$g(t) = \begin{cases} A\,t, & \text{if } 0 \leq t < T \\ 0, & \text{in any other case} \end{cases}.$$

   I) Get the value of A if the pulse needs to be normalized according to the normalization criteria needed for CPM modulations. Explain if the modulation defined with that pulse is a full-response CPM or a partial-response CPM (explaining clearly the difference between both modalities).

II) For the CPM of previous section get the phase tree for two periods labelling carefully the axes and all the transitions if the information sequence is quaternary $I[n] \in \{\pm 1, \pm 3\}$.

Exercise 3.5 Design two modulations based on FSK so that two users can transmit binary information simultaneously without interference and using the smallest bandwidth possible. Give an example of the frequencies that you would use for each user and identify the pulses $g_i(t)$ in each case.

Exercise 3.6 M-ary (with M symbols) frequency modulations CPFSK and MSK make use of pulses with the following shape

$$g_i(t) = \text{sen}\,(\omega_i t)\;\; w_T(t),\;\; \text{para } i = 0, 1, \cdots, M-1,$$

where $w_T(t)$ is a causal squared pulse of unit amplitude and duration T seconds. For a given communication system, the usable range for the frequencies of each pulse is limited between two frequencies, such that $\omega_a \leq \omega_i \leq \omega_b$, where $\omega_a = 2\pi f_a$ and $\omega_b = 2\pi f_b$, with $f_a = 950$ MHz and $f_b = 1250$ MHz.

For the case $M = 4$, obtain the maximum possible symbol rate, and the values of the four frequencies (ω_i or f_i, $i = 0, 1, 2, 3$) when using:

a) A MSK modulation.

b) A CPFSK modulation.

Exercise 3.7 A differential PSK modulator generates the sequence of transmitted symbols in the following way: the symbols of the constellation are

$$A[n] = R\, e^{j\phi[n]}$$

where the phase that is transmitted at instant n, $\phi[n]$, is obtained taking into account the phase transmitted in the previous instant, $\phi[n-1]$, and the phase associated to the new binary information $\Delta_\phi[n] = \left\{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\right\}$ radians

$$\phi[n] = \phi[n-1] + \Delta_\phi[n]$$

a) Plot the block diagram taking into account that the input is the sequence of data bits $B_b[\ell]$ and the output is the complex baseband signal $s(t)$.

b) If the initial reference phase is $\phi[-1] = \frac{\pi}{4}$, obtain the alphabet of transmitted symbols, plot the constellation, and calculate the mean energy per symbol if all symbols are transmitted with the same probability.

c) Provide an optimal binary assignment to minimize the bit error rate (BER).

d) If carriers used in the transmitter to generate the bandpass signal have frequency ω_c rad/s and a null phase, and carriers used at the receiver have the same frequency but a different phase $\theta_c \neq 0$ radians (non-coherent receiver), obtain the received noiseless constellation.

e) Design a proper receiver to deal with the previous situation with a non-coherent receiver (you have to provide the analytical expressions or the block diagram to obtain the estimation of the received bits from the observation $q[n]$).

Exercise 3.8 Some questions related with differente angle modulations:

a) OQPSK ("*Offset Quadrature Phase Shift Keying*") modulation is an angle modulation defined through a modification performed over QPSK modulation. Explain clearly what undesired effect of QPSK is avoided in OQPSK and the modification is necessary to perform in the generation of the modulated signal to obtain this advantage.

b) CPFSK ("*Continuous Phase Frequency Shift Keying*") and MSK ("*Minimum Shift Keying*") are frequency modulations when for an M-ary constellation M pulses of different frequencies are used

$$g_i(t) = \text{sen}(\omega_i t)\ w_T(t),\ \text{para } i \in \{0, 1, \cdots, M-1\},$$

where $w_T(t)$ denotes a causal rectangular window of length T seconds. Explain the conditions that frecuencies ω_i have to satisfy en each modulation.

c) A differential phase shift keying modulation uses the following 4-PSK constellation

$$\boldsymbol{a}_0 = \begin{bmatrix} +1 \\ +1 \end{bmatrix},\ \boldsymbol{a}_1 = \begin{bmatrix} -1 \\ +1 \end{bmatrix},\ \boldsymbol{a}_2 = \begin{bmatrix} -1 \\ -1 \end{bmatrix},\ \boldsymbol{a}_3 = \begin{bmatrix} +1 \\ -1 \end{bmatrix}.$$

Perform an optimal binay assignment for the DPSK system, and obtain the sequence of transmitted symbols, $A[n]$, generated by the following binary information when it is assumed that $A[-1] = \boldsymbol{a}_0$.

m	0	1	2	3	4	5	6	7
$B_b[m]$	0	1	1	0	0	0	1	1

d) Explain the difference between a full response CPM ("*Continuous Phase Modulation*") and a prtial response CPM. For each variant, it have to be clearly stated the parameter that is different, and the range for this parameter in each variant.

Exercise 3.9 Several systems with different angle modulations will be analyzed.

a) A 4-ary mimimum shift keying (MSK) modulation is employed to tramsmit at binary rate R_b=2 Mbits/s, with the constraint that all frequencies associated to the modulation pulses have to satisfy $\omega_i \geq 3\pi$ Mrad/s (or $f_i \geq 1{,}5$ MHz) for $i \in \{0, 1, 2, 3\}$. Obtain the 4 frequencies of the system with the purpose of having the lowest possible values.

b) Repeat the previous question if now a continuous phase frequency shift keying (CPFSK) modulation is used.

c) Consider now a differential phase shift keying modulation using a QPSK constellation with normalized levels

$$\boldsymbol{a}_0 = \begin{bmatrix} +1 \\ +1 \end{bmatrix},\ \boldsymbol{a}_1 = \begin{bmatrix} -1 \\ +1 \end{bmatrix},\ \boldsymbol{a}_2 = \begin{bmatrix} -1 \\ -1 \end{bmatrix},\ \boldsymbol{a}_3 = \begin{bmatrix} +1 \\ -1 \end{bmatrix}.$$

Design the binary assignment of the DPSK system and, assuming that previous symbol (reference symbol) is $A[-1] = \boldsymbol{a}_0$, obtain the symbol sequence $A[n]$ that is produced by the following bit sequence

m	0	1	2	3	4	5	6	7
$B[m]$	0	1	1	1	1	0	0	0

Exercise 3.10 Several angle modulations are considered

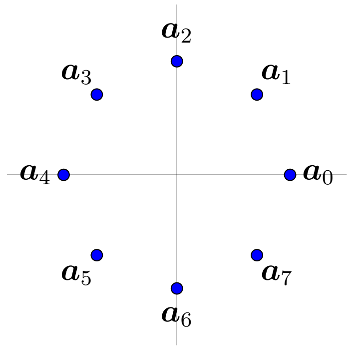
a) In a continuous phase modulation (CPM), explain the difference between a partial-response and a full-response modulation, identifying the differential feature for each variant, and provide an illustrative example for each one of them.

b) Explain how phase continuity is obtained, and write the conditions that the frequencies of the different pulses that are used have to satisfy in the following frequency modulations:

i) Continuous phase frequency shift keying (CPFSK) modulation.

ii) Minimum shift keying (MSK) modulation.

c) A phase modulation employs a 8-PSK constellation with symbols



Provide an appropriate binary assignment in the following cases:

i) A conventional phase shift keying (PSK) modulation is used.

ii) A differential phase shift keying (DPSK) modulation is used.

Exercise 3.11 A frequency modulation employs 8 frequencies to transmit digital information. Discuss if the 8 frequencies are valid for a CPFSK modulation and/or for a MSK modulation, explaining clearly the reason for each modulation, and in the case of a positive answer obtain the symbol rate and the binary rate for each modulation when

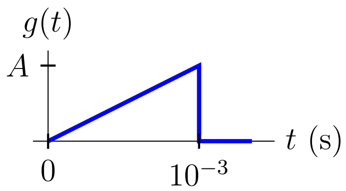
a) The 8 frequencies are:

f_1 (kHz)	f_2 (kHz)	f_3 (kHz)	f_4 (kHz)	f_5 (kHz)	f_6 (kHz)	f_7 (kHz)	f_8 (kHz)
1500	1600	1700	1800	1900	2000	2100	2200

b) The 8 frequencies are:

f_1 (kHz)	f_2 (kHz)	f_3 (kHz)	f_4 (kHz)	f_5 (kHz)	f_6 (kHz)	f_7 (kHz)	f_8 (kHz)
1500	1700	1900	2100	2300	2500	2700	2900

Exercise 3.12 A 4-ary, $I[n] \in \{\pm 1, \pm 3\}$, continuous phase modulation, or CPM, with modulation index $h = 2$ uses the normalized transmitter filter that is shown below to transmit at a binary rate of 2 kbits/s.



a) Explain the difference among a partial-response CPM and a full-response CPM, and identify the class of CPM modulation for this system.

b) Obtain the value of amplitude constant A.

c) Plot the phase tree for two symbol intervals.