Chapter 3 : Solutions of the Exercises

Exercise 3.1 (Solution) a) The modulation for each weveform is

- waveform A: CPFSK
- waveform B: QPSK
- waveform C: MSK
- waveform D: OQPSK

b) Transmitted data sequence is, in both cases

$$I[0] = -1,\ I[1] = +1,\ I[2] = -1,\ I[3] = +1,\ I[4] = +1,\ I[5] = -1.$$

Exercise 3.2 (Solution) a) Pulse amplitude is

$$A = \frac{3}{4T}.$$

It is a full-response CPM because duration of the transmitter pulse is limited to a single symbol interval (T seconds).

b) Phase tree is shown in Fig. 3.1.

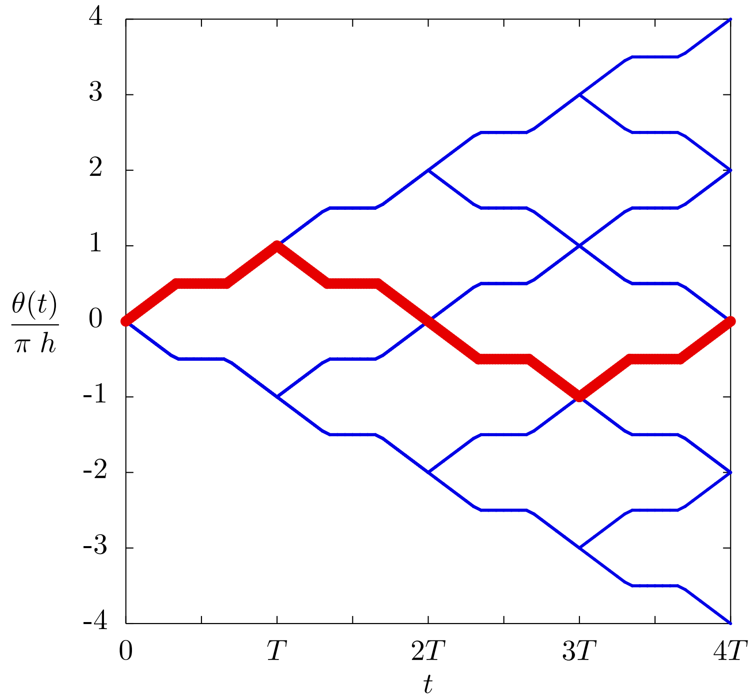
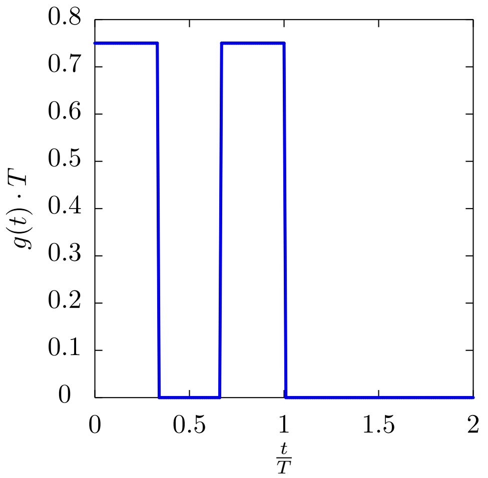


Figura 3.1: Phase tree for the CPM modulation.

Exercise 3.3 (Solution) a) For the first system

i) $I[n] \in \{\pm 1\}$.

ii) Pulse $g(t)$



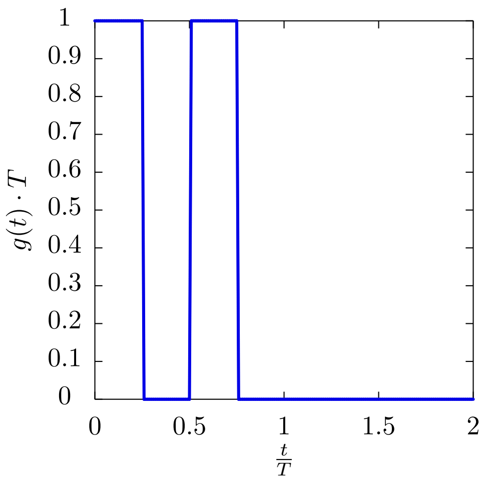
iii) Data sequence

$$I[0] = +1,\ I[1] = -1,\ I[2] = +1,\ I[3] = +1.$$

b) For the second system

i) Now $I[n] \in \{\pm 1, \pm 3\}$.

ii) Pulse $g(t)$



iii) Data sequence

$$I[0] = +3,\ I[1] = +1,\ I[2] = -1.$$

Exercise 3.4 (Solution) a) The minimum shift is

i) For a CPFSK modulation

$$\Delta_\omega = \omega_i - \omega_{i-1} = \frac{2\pi}{T} \text{ rad/s}, \quad \Delta_f = f_i - f_{i-1} = R_s \text{ Hz}.$$

ii) For a MSK modulation

$$\Delta_\omega = \omega_i - \omega_{i-1} = \frac{\pi}{T} \text{ rad/s}, \quad \Delta_f = f_i - f_{i-1} = \frac{R_s}{2} \text{ Hz}.$$

b) The 180º phase shifts happen when a simultaneous change happens in both the in-phase, $s_I(t)$ and the quadrature, $s_Q(t)$, components of the baseband signal. To avoid this circunstance, a delay of half of the symbol length is introduced in one of the components, typically in the quadrature component

$$s_I(t) = \sum_n A_I[n]\; g(t - nT)$$

$$s_Q(t) = \sum_n A_Q[n]\; g\left(t - nT - \frac{T}{2}\right).$$

c) In this case a CPM modulation is considered

   i) It is a full-response modulation, because the transmitter filter $g(t)$ has a duration of a symbol length (in a partial-response CPM, the duration of $g(t)$ is higher than T, typically it last L symbol intervals).

   The value for A is

$$A = \frac{1}{T^2}.$$

   ii) Phase tree is plotted in Figure 3.2.

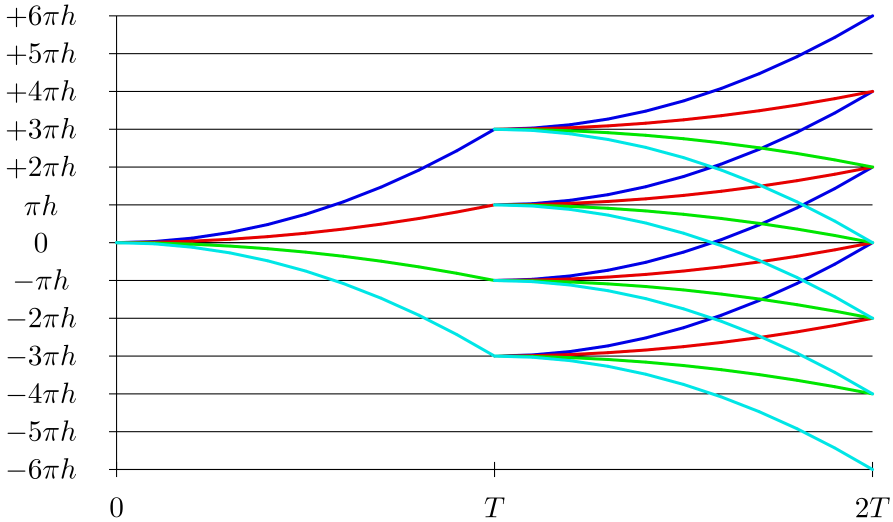


Figura 3.2: Phase tree.

Exercise 3.5 (Solution) In this case, each user has $M = 2$ symbols (binary), which means that will use two frequencies. In the case of a CPFSK, frequencies have to be consecutive multiples of $\frac{2\pi}{T}$ rad/s (or R_s Hz). If we have two users, there are four frequencies, i.e. for a given integer K

$$\omega_{0,u1} = \frac{2\pi}{T} \times K,\; \omega_{1,u1} = \frac{2\pi}{T} \times (K+1),\; \omega_{0,u2} = \frac{2\pi}{T} \times (K+2),\; \omega_{1,u2} = \frac{2\pi}{T} \times (K+3).$$

Corresponding pulses are

$$g_{i,uk}(t) = \sin\left(\omega_{i,uk} t\right)\; w_T(t),$$

where $w_T(t)$ is a causal window of length T seconds. For instance, frequencies can be

$$\omega_{0,u1} = \frac{2\pi}{T},\; \omega_{1,u1} = \frac{4\pi}{T},\; \omega_{0,u2} = \frac{6\pi}{T},\; \omega_{1,u2} = \frac{8\pi}{T}.$$

Exercise 3.6 (Solution) a) For a MSK modulation maximum symbol rate is

$$R_s = 200 \text{ Mbauds.}$$

The 4 frequencies are

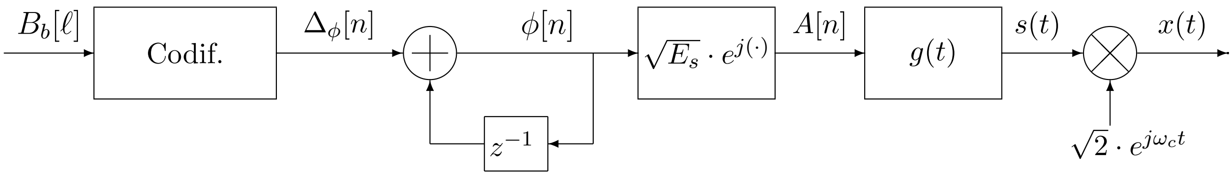
$$f_0 = 950 \text{ MHz}, \; f_1 = 1050 \text{ MHz}, \; f_2 = 1150 \text{ MHz}, \; f_3 = 1250 \text{ MHz}.$$

b) For a CPFSK modulation

$$R_s = 96{,}15 \text{ Mbauds.}$$

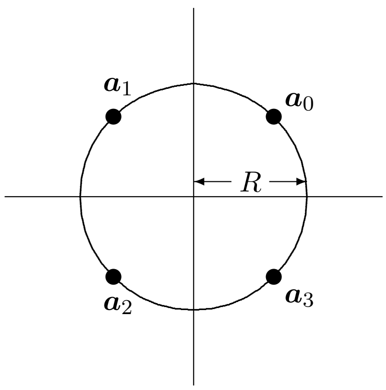
Frequencies are $f_0 = 961{,}54$ MHz, $f_1 = 1057{,}70$ MHz, $f_2 = 1153{,}85$ MHz, $f_3 = 1250$ MHz.

Exercise 3.7 (Solution) a) Block diagram of the transmitter is



The exercise only asked to plot the system up to the baseband complex signal $s(t)$, but picture includes band-pass modulation to generate signal $x(t)$.

b) With this initial reference phaze, modulation is a 4-QAM, with phases $\phi[n] \in \left\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\right\}$ radians, as shown in the picture



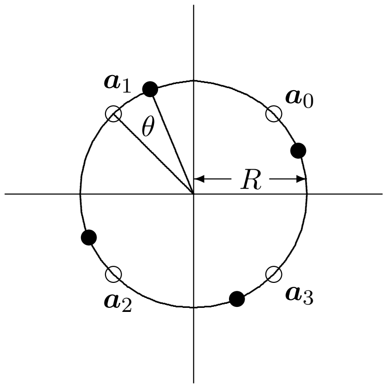
Mean energy per symbol is

$$E_s = \sum_{i=0}^{M-1} p_A(\boldsymbol{a}_i)\, \mathcal{E}\{\boldsymbol{a}_i\} = \sum_{i=0}^{M-1} p_A(\boldsymbol{a}_i)\, ||\boldsymbol{a}_i||^2 = R^2.$$

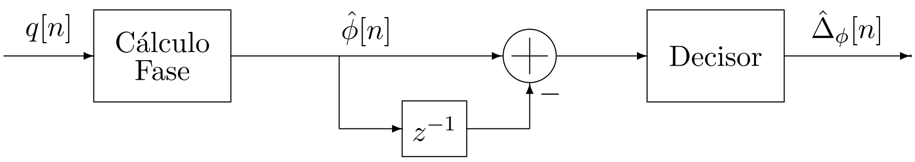
c) An example can be

$\Delta_\phi[n]$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
bits	00	10	11	01

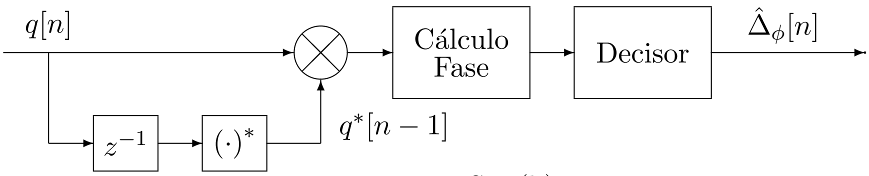
d) The effect is a rotation in the received constellation, $-\theta$ radians. Equivalent discrete channel is in this case $p[n] = e^{-j\theta}\delta[n]$. An example for $\theta = \frac{\pi}{8}$ rad is shown below.



e) The receiver has to estimate the difference between phases of current symbol and previous symbol. Several opcions are shown in the figure



Receptor DPSK (a)



Receptor DPSK (b)

Exercise 3.8 (Solution) a) In an OQPSK modulation the goal is to avoid 180º phase shifts appearing in a QPSK modulation. This can be achieved by introducing a delay of half a symbol period in the quadrature component.

b) For a CPFSK modulation, frequencies have to be consecutive integer multiples of $\frac{2\pi}{T}$ rad/s (or R_s Hz). Therefore, frequency separation is $\Delta_\omega = \omega_{i+1} - \omega_i = \frac{2\pi}{T}$ rad/s (or $\Delta_f = f_{i+1} - f_i = R_s$ Hz). Therefore, frequencies can be written as

$$\omega_0 = \frac{2\pi}{T} \times K,\ \omega_1 = \frac{2\pi}{T} \times (K+1),\ \omega_2 = \frac{2\pi}{T} \times (K+2),\ \cdots,\ \omega_{M-1} = \frac{2\pi}{T} \times (K+M-1),$$

in rad/s, with K being an integer.

For a MSK modulation, the only condition is that separation between consecutive frequencies has to be $\Delta_\omega = \omega_{i+1} - \omega_i = \frac{\pi}{T}$ rad/s (or $\Delta_f = f_{i+1} - f_i = \frac{R_s}{2}$ Hz). Therefore, frequencies are as follows

$$\omega_0 = A,\ \omega_1 = A + 1 \times \frac{\pi}{T},\ \omega_2 = A + 2 \times \frac{\pi}{T},\ \cdots,\ \omega_{M-1} = A + (M-1) \times \frac{\pi}{T},$$

where A can be any arbitrary angular frequency in rad/s.

c) In a phase modulation, a symbol is identified by its pahse, because $A[n] = \sqrt{E_s}\, e^{j\phi[n]}$. In this case

Símbolo $A[n]$	a_0	a_1	a_2	a_3
$\phi[n]$ (rad/s | degrees)	$\frac{\pi}{4}$ | 45	$\frac{3\pi}{4}$ | 135	$\frac{5\pi}{4}$ | 225	$\frac{7\pi}{4}$ | 315

In a differential phase modulation, the key idea is that the phase of the symbol that is transmitted at a given discrete instant n is obtained incrementaly with respect to the phase of the previous symbol

$$\phi[n] = \phi[n-1] + \Delta_\phi[n].$$

Therefore, binary assignment is done for this pahse increment $\Delta_\phi[n]$, using Gray encoding. An example of such asignment (not the only possible) is

$\Delta_\phi[n]$ (rad/s | degrees)	$0 \mid 0$	$\frac{\pi}{2} \mid 90$	$\pi \mid 180$	$\frac{3\pi}{2} \mid 270$
bits	00	01	11	10

Therefore, taking into account that $A[-1] = \boldsymbol{a}_0$ (i.e., $\phi[-1] = \frac{\pi}{4}$), transmitted sequence is

n	0	1	2	3
bits	01	10	00	11
$\Delta_\phi[n]$ (rad/s | grados)	$\frac{\pi}{2} \mid 90$	$\frac{3\pi}{2} \mid 270$	$0 \mid 0$	$\pi \mid 180$
$\phi[n]$ (rad/s | grados)	$\frac{3\pi}{4} \mid 135$	$\frac{\pi}{4} \mid 45$	$\frac{\pi}{4} \mid 45$	$\frac{5\pi}{4} \mid 225$
Símbolo $A[n]$	$\boldsymbol{a}_1$	$\boldsymbol{a}_0$	$\boldsymbol{a}_0$	$\boldsymbol{a}_2$

d) The difference between both variants is the duration of transmitter filter. In a full-response CPM, this duration is limited to a single symbol interval (T second). In a partial-response CPM, duration is limited to de duration of L symbols (LT seconds), with $L > 1$.

Exercise 3.9 (Solution) a) In the case of a MSK modulation

$$f_0 = 1{,}5 \text{ MHz},\ f_1 = 2{,}0 \text{ MHz},\ f_2 = 2{,}5 \text{ MHz},\ f_3 = 3{,}0 \text{ MHz}.$$

b) For a CPFSK modulation

$$f_0 = 2 \text{ MHz},\ f_1 = 3 \text{ MHz},\ f_2 = 4 \text{ MHz},\ f_3 = 5 \text{ MHz}.$$

c) An example (it is not the only possible choice) is

$\Delta_\phi[n]$ (rad/s | degrees)	$0 \mid 0$	$\frac{\pi}{2} \mid 90$	$\pi \mid 180$	$\frac{3\pi}{2} \mid 270$
bits	00	01	11	10

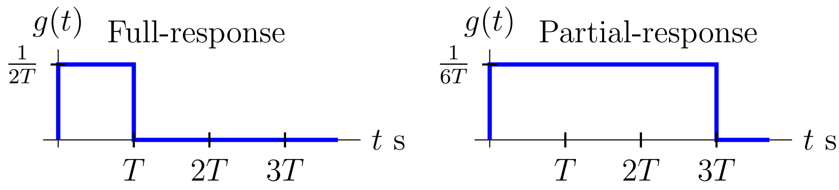
Using this particular assignment, transmitted data sequence is shown in the table below

n	0	1	2	3
bits	01	11	10	00
$\Delta_\phi[n]$ (rad/s | grados)	$\frac{\pi}{2} \mid 90$	$\pi \mid 180$	$\frac{3\pi}{2} \mid 270$	$0 \mid 0$
$\phi[n]$ (rad/s | grados)	$\frac{3\pi}{4} \mid 135$	$\frac{7\pi}{4} \mid 315$	$\frac{5\pi}{4} \mid 225$	$\frac{5\pi}{4} \mid 225$
Símbolo $A[n]$	$\boldsymbol{a}_1$	$\boldsymbol{a}_3$	$\boldsymbol{a}_2$	$\boldsymbol{a}_2$

Exercise 3.10 (Solution) a) The difference is related with the length of the trasmitter filter $g(t)$:

- In a full-response modulation, duration is limited to a single symbol interval, in this case $T = 1/R_s$ seconds for a transmission rate of R_s bauds.
- In a partial-response modulation, the duration is higher than a symbol interval, typically L intervals (duration $L \times T$ seconds, with $L \in \mathbb{Z} > 1$).

Two examples, using squared pulses, is shown in the figure



b) For the frequency modulations

I) In a CPFSK modulation, the condition is that frequencies have to be consecutive integer multiples of $\frac{2\pi}{T}$ rad/s, or of R_s Hz. With these frequencies, there is an integer number of cycles in T seconds, which guaranties the phase continuity (final phase is equal to the initial phase for all pulses).

II) In a MSK modulation, the condition is that the separison between two consecutive frequencies has to be $\frac{\pi}{T}$ rad/s, or $\frac{R_s}{2}$ Hz. Phase continuity is obtained by introducing memory to recursively compute the initial phase, $\theta[n]$, for each transmitted symbol. The analitycal expression to recursively compute this initial phase is

$$\theta[n] = \theta[n-1] + \frac{\pi n}{2}\left(I[n-1] - I[n]\right)$$

c) An example of binary assignment for each modulation is

I) For the PSK

Símbolo	$\boldsymbol{a}_0$	$\boldsymbol{a}_1$	$\boldsymbol{a}_2$	$\boldsymbol{a}_3$	$\boldsymbol{a}_4$	$\boldsymbol{a}_5$	$\boldsymbol{a}_6$	$\boldsymbol{a}_7$
$\phi[n]$	0º	45º	90º	135º	180º	225º	270º	315º
bits	000	100	110	010	011	111	101	001

II) For the DPSK

$\Delta_\phi[n]$	0º	45º	90º	135º	180º	225º	270º	315º
bits	000	100	110	010	011	111	101	001

and now the phase of the transmitted symbols is obtained incrementally

$$\phi[n] = \phi[n-1] + \Delta_\phi[n].$$

Exercise 3.11 (Solución) a) In the first case

- CPFSK: frequencies are valid, $R_s = 100$ kbauds, $R_b = 300$ kbits/s.
- MSK: frequencies arevalid, $R_s = 200$ kbauds, $R_b = 600$ kbits/s.

b) In the second case

- CPFSK: frequencies are not valid.
- MSK: frecuencias válidas, $R_s = 400$ kbaudios, $R_b = 1200$ kbits/s.

Exercise 3.12 (Solution) a) In a full response CPM, the response of the transmitter filter has a duration lower or equal to the symbol lenght, and in a partial response CPM the duration is higher than the symbol length (in general, the duration is L symbol intervals). Therefore, the modulation is a full response CPM.

b) Amplitude is $A = 10^3$ V.

c) The phase tree is plotted below