Chapter 5 : Exercises

Exercise 5.1 The encoding matrix for a linear block code $(4,8)$ is given next. Compute the coding rate, the minimum distance and the syndrome table.

$$\mathbf{G} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

Exercise 5.2 A convolutional code has the following generator matrix

$$\mathbf{G}(D) = \begin{bmatrix} 1 & D & D+1 \\ D^2 & 1 & D^2+D+1 \end{bmatrix}$$

a) Obtain the coding rate of the code.

b) Plot the schematic representation of the code.

c) Plor the trellis diagram for the code.

d) Obtain the minumum Hamming distance, D_{min}^{H} for the code.

Exercise 5.3 A convolutional code with coding rate $1/2$ has the following generator matrix:

$$\mathbf{G}(D) = \begin{bmatrix} D^2+1 & D^2+D+1 \end{bmatrix}.$$

Transmitter uses a BPSK (or 2-PAM) modulation with normalized levels $A[n] \in \pm 1$. Binary assignment is $B[n] = 0$ for $A[n] = -1$ and $B[n] = 1$ for $A[n] = +1$. The system transmit a cyclic header of 2 zeros between each block of 6 data bits to reset the state of the convolutional. Decode, using the Viterbi algorithm with soft and hard outputs the following received sequence

$$\begin{array}{lcccccccc} q^{(0)}[n]: & +3{,}06 & -0{,}70 & -0{,}58 & -1{,}37 & -0{,}82 & -2{,}63 & -1{,}37 & -0{,}85 \\ q^{(1)}[n]: & +1{,}08 & -1{,}06 & -2{,}89 & +0{,}33 & +1{,}92 & -1{,}64 & -0{,}70 & +2{,}30 \end{array}$$

NOTE: For decoding with hard output you need first to get the hard decision on the given sequence.

Exercise 5.4 Two channel codes are going to be evaluated in a digital communication system: a linear block code and a convolutional code.

a) The generator matrix of the linear block code is

$$\mathbf{G} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}.$$

I) Get the code minimum distance.

II) Transform $\mathbf{G}$ in a systematic matrix $\mathbf{G}'$ that could be used to obtain a parity check matrix allowing to define a syndrome table for the code.

III) Get the parity-check matrix.

IV) Get the syndrome table.

b) Next, consider the convolutional code given by the generator matrix

$$\mathbf{G} = \begin{bmatrix} (1+D) & D & 1 & (1+D) \\ D & (1+D) & 1 & 1 \end{bmatrix}.$$

I) Get the shematic representation of the encoder.

II) Plot the code trellis diagram.

III) Obtain the code minimum distance, D_{min}.

IV) Get, assuming as starting and ending state the zero state, i.e. $\psi_0 = [0, 0, \cdots, 0]$, the decoded message when the received sequence is

$$\mathbf{r} = [1011000110100110].$$

c) Compare both system performance if the underlying BSC channel has a bit error rate ε.

Exercise 5.5 Two channel codes will be analyzed

a) A linear block code has the following generator matrix

$$\boldsymbol{G} = \begin{bmatrix} 0 & 1 & a & 0 & b \\ c & d & 1 & 1 & 1 \end{bmatrix}$$

I) Get a, b, c and d values to obtain the maximum detection and correction capabilities.

II) Obtain the syndrome table and decode the following received words.

$$\boldsymbol{r}_0 = [10001],\ \boldsymbol{r}_1 = [10011],\ \boldsymbol{r}_2 = [11001]$$

b) A convolutional code has the following generator matrix

$$\boldsymbol{G} = \left[1 + D + D^2,\ 1\right].$$

The information data are transmitted with a 4-QAM modulation with the following binary assignment.

Symbol	$+1+j$	$-1-j$	$+1-j$	$-1+j$
Bits	11	00	10	01

I) Get the schematic representation of the encoder, and the trellis diagram.

II) Encode the information sequence $B^{(0)}[\ell] = [101100]$ under the assumption that the starting state is the zero state, ψ_0. Plot the path of the output sequence through the trellis diagram.

III) Get the code performance working both with hard and soft decoding.

IV) Decode the received sequence

$$\boldsymbol{r} = [101001010011],$$

assuming that $B^{(0)}[\ell] = 0$ for $\ell < 0$ and $\ell \geq 4$ (i.e. the initial and final states are ψ_0).

Exercise 5.6 Two linear block codes are given by the following generator matrices:

$$\mathbf{G}_1 = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix} \; \mathbf{G}_2 = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

a) Are the codes systematic?

b) Get the error detecting and correcting capabilities.

c) Choose the best code from previous section, get its syndrome table and decode the following received words:

$$\mathbf{r}_a = [01101] \text{ and } \mathbf{r}_b = [11111].$$

Exercise 5.7 A convolutional code has the following generator matrix:

$$\mathbf{G}(D) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & D & D+1 \end{bmatrix}$$

a) Obtain the code rate and get the schematic representation of the encoder.

b) Obtain the state diagram, the trellis diagram and get the minimum distance of the code.

c) Assuming the starting state the zero state, encode the following input sequence

$$B[m] = 11011000.$$

d) Determine if the following sequence 110010111000 is a possible fragment of a codeword. Assume any possible starting state.

e) Consider now a simplified version of the previous code with generator matrix[1]:

$$\mathbf{G}(D) = \begin{bmatrix} D & D+1 \end{bmatrix}$$

Assuming the starting and ending state the zero state, find the decoded message when the received message is

$$\mathbf{r} = [10001111011011].$$

Exercise 5.8 For a communication system, three different codes $\mathcal{C}_1$, $\mathcal{C}_2$ y $\mathcal{C}_3$ are defined. The corresponding codewords are the following

$$\begin{aligned} \mathcal{C}_1 &= \{01, 10\}, \\ \mathcal{C}_2 &= \{00000, 01010\}, \\ \mathcal{C}_3 &= \{00000, 10100, 01111, 11011\}. \end{aligned}$$

a) Obtain for each code the parameters k, n, the coding rate and the minimum Hamming distance.

b) Determine which of this codes are linear and for those get the generator matrix.

c) Find the systematic codes among the three different encoders.

[1]Note that the new code is a simplification of the previous one, obtained removing the first input and the first output.

d) Is it possible to improve the performance of $\mathcal{C}_2$ without modifying the parameters k, n? Explain and discuss the way to obtain the improvement.

e) If the received sequence is $\mathbf{r} = [11111]$, get the word with maximum likelihood between the set of possible transmitted codewords. Explain the procedure used to obtain the transmitted codeword with highest likelihood.

Exercise 5.9 We want to design a communication system with a channel code with rate 1/2. There are two possibilities:

- A linear block code with the following generator matrix:

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

- A convolutional code with the following generator matrix:

$$\boldsymbol{G}(D) = [D,\ 1 + D + D^2].$$

In both cases, after the encoder, a 2-PAM (o BPSK) modulation is used, with a distance between symbols in the constellation d_{min}^{BPSK}.

a) Obtain the set of codewords of the block codes, the parity-check matrix and the minimum distance.

b) Compute the probability of error with the block encoder using a "hard" detector. Provide an expression for this probability as function of the minimum distance of the code and the distance in the constellation d_{min}^{BPSK}.

c) Plot the trellis diagram of the convolutional code.

d) Calculate the probability of error with the convolution encoder using both "soft" and "hard" observations, in this case assuming $d_{min}^{BPSK} = 1$.

e) Find the transmitted sequence with the block encoder for the message $B_b[\ell] = [100110]$. Then, assuming that the channel produces errors in the first, sixth and ninth bits, obtain the recovered sequence at the output of the decoder.

f) Find the transmitted sequence with the convolutional encoder for the message $B_b[\ell] = [100]$ between the corresponding headers to guarantee that the starting and ending states are the zero state, and the channel produces an error in the first bit. Obtain the sequence at the output of the decoder using a "hard" decision criterion.

Exercise 5.10 A convolutional code has the following generator matrix

$$\mathbf{G}(D) = [1,\ 1 + D,\ 1 + D + D^2].$$

a) Obtain the schematic representation and the trellis diagram for this encoder.

b) Discuss if the code is systematic or not.

c) Assume that the received sequence is

$$\mathbf{r} = [111\ 111\ 110\ 011\ 001],$$

and assume also the starting and ending states are the zero state, which is forzed by means of the transmission of the appropriate number of zeros. Determine the most likelihood transmitted sequence and the corresponding message.

Exercise 5.11 In a digital communication system we decide to improve the error correction capability of a repetition block code with generator matrix $\mathbf{G} = [1\ 1\ 1]$. For that we concatenate the output of the repetition code to the input of a convolutional code with generator matrix

$$\mathbf{G} = \begin{bmatrix} 1 & D & 0 & 0 \\ D & 1 & 0 & 0 \\ 0 & 0 & 1+D & D \end{bmatrix}$$

For the block and convolutional encoders separately answer the following questions:

a) Get the codewords of the block code and obtain its minimum distance.

b) Get the block diagram of the convolutional code and its trellis diagram. For the trellis diagram it is not necessary to label all the transitions, only label those that emerge from the state $\psi_0 = [000]$ and those that arrive to the same state $\psi_0 = [000]$. What would be the minimum distance of that code?

If we name the codewords at the output of the block encoder at discrete instant ℓ as $\mathbf{c}[\ell] = [c_0[\ell]\ c_1[\ell]\ c_2[\ell]]$ and $\mathbf{B}[\ell] = [B^{(0)}[\ell]\ B^{(1)}[\ell]\ B^{(2)}[\ell]]$ as the input words to the convolutional encoder at discrete instant ℓ, the concatenation is made by assigning $B^{(i)}[\ell] = c_i[\ell]$ for $i = 0 \ldots 2$. Answer the following questions related to the concatenated code.

a) Get the code rate of the concatenated code. Obtain the trellis diagram arising from the concatenation of both codes and its minimum distance.

b) Compare the error correction capabilities of the concatenated code with each of the component encoders separately. Discuss if the concatenated encoder needs more or less bandwidth than the block code to maintain the bit rate. Discuss similarly for the convolutional code.

c) From the trellis diagram obtained for the concatenated code get a block diagram and the generator matrix in D polynomials of a convolutional encoder that behaves exactly as the concatenated code.

Exercise 5.12 Two block codes are given with the following assignments between uncoded block bits, $\mathbf{b}_i$, and codewords $\mathbf{c}_i$.

i	$\mathbf{b}_i$	$\mathbf{c}_i$
0	0 0	0 0 0 0 0
1	0 1	1 1 0 1 0
2	1 0	1 0 1 1 1
3	1 1	0 1 1 0 1

Code A

i	$\mathbf{b}_i$	$\mathbf{c}_i$
0	0 0	0 0 0 0 0
1	0 1	1 0 1 0 1
2	1 0	1 1 0 1 0
3	1 1	1 1 1 1 1

Code B

a) For each of the codes:

   I) Explain if the code is linear or not and if it is systematic. Answer should be clearly reasoned.

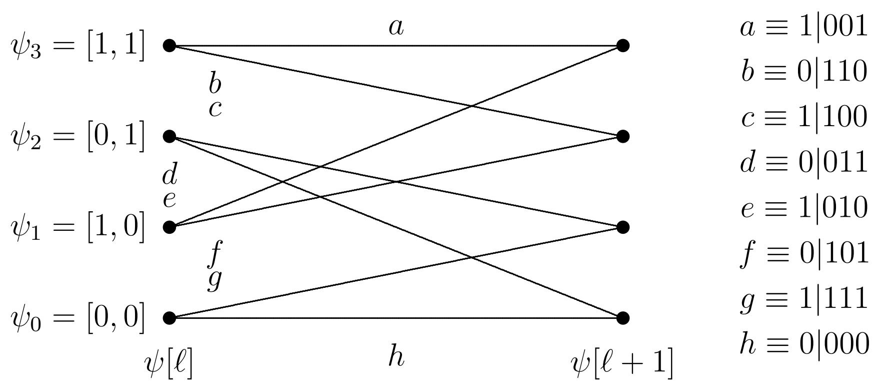
   II) Get the number of errors that each code is able to correct.

b) For the linear code (if both are, consider the one most appropriate, clearly justifying your election), get the generator matrix and the parity check matrix.

c) For the same code, get the syndrome table and decode, providing the estimated uncoded bits $\hat{\mathbf{b}}_i$, the following received words.

$$\mathbf{r_a} = 1\ 1\ 1\ 0\ 1,\ \mathbf{r_b} = 1\ 0\ 0\ 1\ 1.$$

Exercise 5.13 A convolutional code, has the following trellis diagram



a) Get the generator matrix and the encoder schematic representation.

b) Assuming that before and after it two headers with the necessary number of zeros are transmitted, encode the following bit information sequence

$$B^{(0)}[0] = 0,\ B^{(0)}[1] = 1,\ B^{(0)}[2] = 1,\ B^{(0)}[3] = 0,\ B^{(0)}[4] = 1,$$

and get the approximate error probability if hard decoding is used.

c) Decode the block of three information bits, $\hat{B}^{(0)}[\ell]$, $\ell \in \{0,1,2\}$, for the following received sequence, assuming that $B^{(0)}[\ell] = 0$ for $n < 0$ and for $n \geq 3$ (headers of zeros)

$$\mathbf{r} = 010\ 111\ 000\ 111\ 011$$

Exercise 5.14 In a digital communication system three linear block codes are available, and in some cases two codes can be concatened to improve the performance of individual codes.

First block code is a repetition code of rate 1/3. Second encoder is a systematic linear block code, systematic by the beginning (the first k bits of the n encoded bits replicate the k uncoded bits with information), whose encoded words of n bits are

$$\mathcal{C}_2 = \{0000, 1001, 0101, 0011, 1100, 1010, 0110, 1111\}.$$

Third linear block code has the following parity check matrix:

$$\mathbf{H}_3 = \left[\begin{array}{ccc|cc} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{array}\right]$$

.

a) Obtain the generator matrices for the three codes.

b) Obtain the detection and correction capabilities for code 2 and for code 3, individually, and discuss which one is better in terms of its correction capability.

c) Obtain the resulting encoded words for the concatanation of codes 1 and 2 (in this order), and provide the rate parameters (parameters k and n) for this concatenated code. Does this concatenation provide any advantage over the individual codes?

d) Obtain the generator matrix for the concatenated code obtained by the concatenation of codes 2 and 3 (in this order), and the rate parameters (parameters k and n) for this concatenated code. Compare the performance of this code with the one of the previous section.

e) Obtain the syndrome table for code 3, and decode, by providing the k uncoded information bits, explaining clearly each step of the syndrome based decoding technique, the following received word:

$$\boldsymbol{r} = 11111.$$

Exercise 5.15 A linear block code has the following dictionary

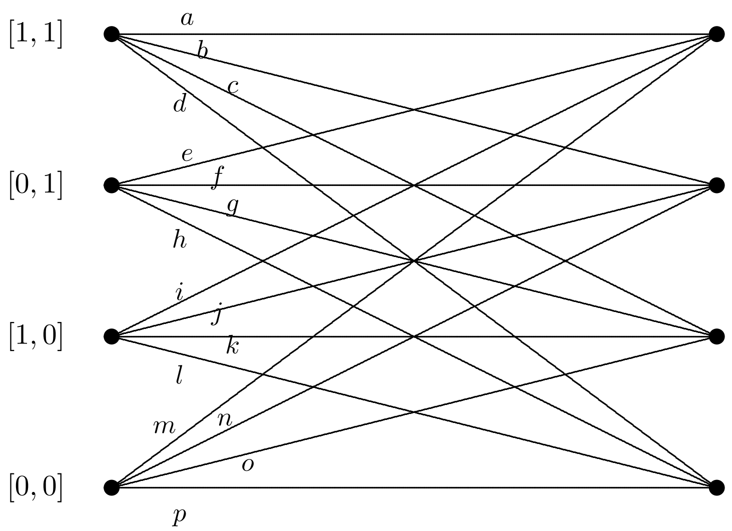
i	$\mathbf{b}_i$	$\mathbf{c}_i$
0	0 0 0	x x x x x x
1	0 0 1	1 1 1 0 0 0
2	0 1 0	x x x x x x
3	0 1 1	1 0 0 1 1 0
4	1 0 0	1 0 1 1 0 1
5	1 0 1	0 1 0 1 0 1
6	1 1 0	x x x x x x
7	1 1 1	0 0 1 0 1 1

a) Obtain coded words $\mathbf{c}_0$, $\mathbf{c}_2$ and $\mathbf{c}_6$ if the code is linear, say if the code is or not systematic explaining why, and calculate the detection and correction capabilities of the channel code.

b) Obtain the generator matrix and the parity check matrix of the code.

c) Obtain the syndrome table, and decode (providing the estimation of the uncoded word $\mathbf{b}_i$), indicating each step of the syndrome based decoding algorithm, if the received word is $\mathbf{r} = 1\ 1\ 1\ 0\ 1\ 1$.

Exercise 5.16 A convolutional encoder has the following trellis diagram



$a \equiv 11|1001$
$b \equiv 01|0011$
$c \equiv 10|1100$
$d \equiv 00|0110$

$e \equiv 11|1101$
$f \equiv 01|0111$
$g \equiv 10|1000$
$h \equiv 00|0010$

$i \equiv 11|1011$
$j \equiv 01|0001$
$k \equiv 10|1110$
$l \equiv 00|0100$

$m \equiv 11|1111$
$n \equiv 01|0101$
$o \equiv 10|1010$
$p \equiv 00|0000$

a) Obtain the generator matrix with D polynomials, and plot the schematic representation of the encoder.

b) Assuming that all the previously transmitted bits are zeros, encode the following binary sequence 10110100 and calculate the approximated probability of error if hard decoding is used and the bit error rate of the modulation used to transmit is $BER = \varepsilon$.

c) Decode, by applying the optimal decoding algorithm (clear evidence of its application has to be provided), the first four information bits, $\hat{B}[m]$, $m \in \{0, 1, 2, 3\}$, for the following received sequence, assuming that $B[m] = 0$ for $m < 0$ and for $m \geq 4$

$$\mathbf{r} = 101100011010$$

Exercise 5.17 Two convolutional codes are considered. First one, $\mathcal{C}_1$, has the following generator matrix

$$\mathbf{G}(D) = [1,\ D]$$

Second one, $\mathcal{C}_2$, has the following generator matrix

$$\mathbf{G}(D) = \begin{bmatrix} 1, & 1+D, & 0, & 0 \\ 0, & 0, & 1, & 1+D \end{bmatrix}$$

a) Obtain the approximated probability of error of codes $\mathcal{C}_1$ and $\mathcal{C}_2$, respectively, working with hard output.

b) Obtain the trellis diagram of the concatenation of both codes, $\mathcal{C}_1$-$\mathcal{C}_2$.

Exercise 5.18 A block code $\mathcal{C}$ is obtained by the concatenation of two linear block codes, $\mathcal{C}_1$ and $\mathcal{C}_2$. Generator matrix for code $\mathcal{C}_1$ dictionary for code $\mathcal{C}_2$ are, respectively

$$\mathbf{G}_1 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix},$$

$\mathbf{b}_i(\mathcal{C}_2)$	$\mathbf{c}_i(\mathcal{C}_2)$	$\mathbf{b}_i(\mathcal{C}_2)$	$\mathbf{c}_i(\mathcal{C}_2)$
000	00000	100	01010
001	00101	101	01111
010	10011	110	11001
011	10110	111	11100

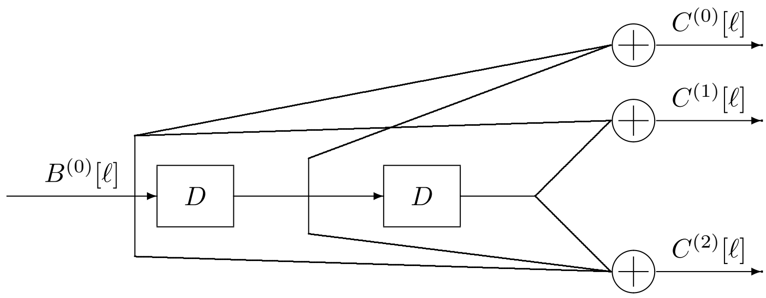
a) Obtain the generator matrix for code $\mathcal{C}_2$, and for each individual code, $\mathcal{C}_1$ and $\mathcal{C}_2$, obtain the number of error that the code can correct explaining clearly how this number is obtained for each code.

b) Say if codes $\mathcal{C}_1$ and $\mathcal{C}_2$ are or not systematic, explaining why.

c) Obtain the dictionary and the generator matrix for the concatenated code $\mathcal{C}$, the number of error that the code is able to correct, and say if the code is or is not systematic explaining why.

d) Obtain the parity check matrix for the concatenated code and the syndrome table that has to be used to obtain the best possible performance.

Exercise 5.19 A convolutional code has the following schematic representation



a) Obtain the generator matrix, the trellis diagram and the approximated performance of the code working with hard output if the digital communication system transmits coded bits with a bit error rate $BER = 10^{-4}$.

b) Obtain the estimated transmitted binary sequence $\hat{B}^{(0)}[\ell]$ for $\ell \in \{0, 1, 2\}$, if it is assumed that the data bits are transmitted in blocks of 3 bits with a header of 2 zeros between each data block to reset the state of the convolutional encoder (i.e., you can assume that $B^{(0)}[-2] = B^{(0)}[-1] = B^{(0)}[3] = B^{(0)}[4] = 0$). The received bit sequence is

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$C[n]$	1	0	1	1	1	1	0	1	0	1	0	0	0	1	0

To obtain the estimation, the optimal decoding algorithm has to be applied, and clear evidence of the application of this decoding algorithm has to be provided.

Exercise 5.20 A linear block code has the dictionary that is given in these tables

$\mathbf{b}_i$	$\mathbf{c}_i$
0 0 0 0	0 0 0 0 0 0 0
0 0 0 1	0 1 1 1 1 0 0
0 0 1 0	1 0 1 1 0 1 0
0 0 1 1	1 1 0 0 1 1 0
0 1 0 0	1 1 1 0 0 0 0
0 1 0 1	1 0 0 1 1 0 0
0 1 1 0	0 1 0 1 0 1 0
0 1 1 1	0 0 1 0 1 1 0

$\mathbf{b}_i$	$\mathbf{c}_i$
1 0 0 0	1 1 1 1 1 1 1
1 0 0 1	1 0 0 0 0 1 1
1 0 1 0	0 1 0 0 1 0 1
1 0 1 1	0 0 1 1 0 0 1
1 1 0 0	0 0 0 1 1 1 1
1 1 0 1	0 1 1 0 0 1 1
1 1 1 0	1 0 1 0 1 0 1
1 1 1 1	1 1 0 1 0 0 1

a) Obtain the following parameters for that code:

- Coding rate and generator matrix.

- Minimum Hamming distance, explaing clearly how it was obtained, and the number of errors that the code is able to correct working with hard output.
- Discuss if the code is perfect or not, explaing clearly the reason.

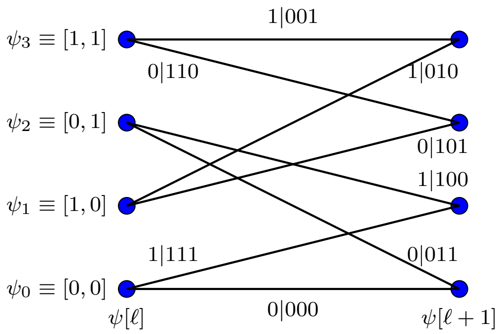
b) Obtain the parity check matrix and the syndrome table.

c) Using the syndrome based decoding technique, enumerating each step, decode the following received word

$$\mathbf{r} = 0\ 1\ 1\ 1\ 0\ 1\ 1$$

Exercise 5.21 Two convolutional encoders are available. For the first one, its generator matrix is know, and for the second one the trellis diagram is provided, which are the ones shown below

$$\mathbf{G} = \begin{bmatrix} 1+D^2 & D & 1 \\ D & 1+D & 1 \end{bmatrix}$$



a) For the first encoder, obtain the schematic representation and plot the trellis diagram partialy, drawing only the branches going out of the states $\psi[\ell]$ all zeros and all ones, respectively, and arriving at the corresponding states $\psi[\ell+1]$.

b) For the second encoder, obtain the schematic representation and its generator matrix.

c) For the second encoder, decode the bits $B^{(0)}[0]$, $B^{(0)}[1]$ and $B^{(0)}[2]$ assuming that headers of zeros are transmitted before and after these 3 bits, if the sequence of received bits is

m	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$R[m]$	0	1	0	1	1	1	1	0	0	1	0	1	0	0	1

REMARK: clear evidence of the application of the optimal algorithm must be provided

Exercise 5.22 A systematic linear block code has a syndrome table that is partially shown below

e	**s**
1 1 0 0 0	1 0 1
1 0 1 0 0	1 1 1
0 0 1 0 0	1 0 0

a) Obtain the parity check matrix and the complete syndrome table.

b) Obtain the following parameters for the code:

- Coding rate.

- Generator matrix.
- Minimum distance of the code, explaining how this distance is obtained.
- Number of errors that the code is able to detect and to correct working with hard output.
- Explain if the code is perfect or not, explaining clearly the reason.
- Obtain the probability of error for this code working in a system that transmits a 2-PAM constellation through an ideal channel with Gaussian noise with power spectral density $N_0/2$.

c) Using the syndrome based decoding technique, and itemizing every step, decode the following received word

$$\mathbf{r} = 1\ 1\ 1\ 1\ 1$$

Exercise 5.23 A convolutional encoder has the following generator matrix

$$\mathbf{G}(D) = [1 + D \qquad 1 + D^2 \qquad 1 + D + D^2]$$

a) Obtain the schematic representation and the trellis diagram of the encoder.

b) Obtain the performance of the convolutional encoder working in a system that transmits a 2-PAM constellation through an ideal channel with Gaussian noise with power spectral density $N_0/2$.

c) Decode the bits $B^{(0)}[0]$, $B^{(0)}[1]$ and $B^{(0)}[2]$ applying the optimal algorithm and assuming that headers with an appropriate number of zeros are transmitted before and after those 3 data bits, if the received binary sequence (hard output) is

m	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$R[m]$	0	1	0	1	1	1	1	0	0	1	0	1	0	0	1

REMARK: clear evidence of the application of the optimal algorithm has to be provided

Exercise 5.24 A linear block code has the following generator matrix

$$\mathbf{G} = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

a) Obtain the following parameters of the code: coding rate, minimum Hamming distance (explaining clearly how it is obtained), number of error that the code is able to detect and able to correct working with hard output.

b) Obtain the parity check matrix and the complete syndrome table.

c) Using syndrome based decoding, and providing evidence of each step, decode the following received word (up to provide the uncoded information bits)

$$\mathbf{r} = 1\ 1\ 1\ 1\ 1\ 0$$

Exercise 5.25 Two convolutional codes are available. For the first one, its generator matrix is known, and for the second one its trellis diagram is provided, both shown below

$$\mathbf{G} = \begin{bmatrix} 1+D & D & 0 \\ 0 & D & 1+D \end{bmatrix}$$

$a_1 \equiv 11|001$, $a_2 \equiv 10|000$, $a_3 \equiv 01|101$, $a_4 \equiv 00|100$

$b_1 \equiv 11|010$, $b_2 \equiv 10|011$, $b_3 \equiv 01|110$, $b_4 \equiv 00|111$

$c_1 \equiv 11|111$, $c_2 \equiv 10|110$, $c_3 \equiv 01|011$, $c_4 \equiv 00|010$

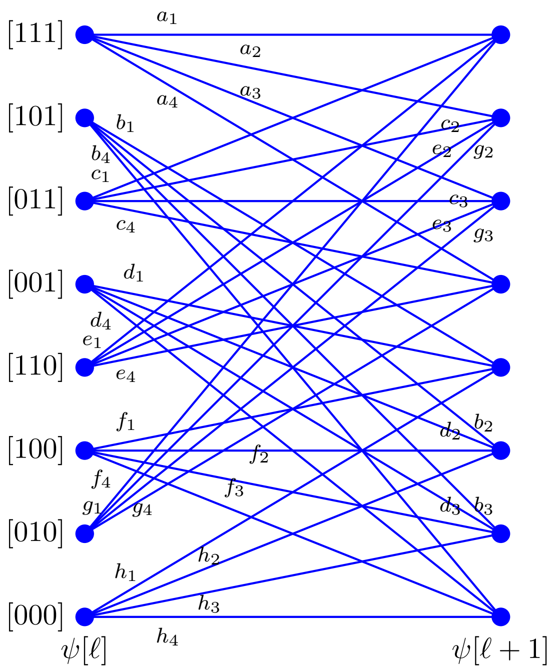
$d_1 \equiv 11|100$, $d_2 \equiv 10|101$, $d_3 \equiv 01|000$, $d_4 \equiv 00|001$

$e_1 \equiv 11|000$, $e_2 \equiv 10|001$, $e_3 \equiv 01|100$, $e_4 \equiv 00|101$

$f_1 \equiv 11|011$, $f_2 \equiv 10|010$, $f_3 \equiv 01|111$, $f_4 \equiv 00|110$

$g_1 \equiv 11|110$, $g_2 \equiv 10|111$, $g_3 \equiv 01|010$, $g_4 \equiv 00|011$

$h_1 \equiv 11|101$, $h_2 \equiv 10|100$, $h_3 \equiv 01|001$, $h_4 \equiv 00|000$



a) Obtain the schematic representation and the trellis diagram for the first encoded (in the trellis, all branches must be drawn, but it is only necessary to include the label for branches going out of the all zeros and all ones states).

b) Obtain the schematic representation and the generator matrix for the second encoder.

c) For the second encoder, and assuming that previously the header with zeros that is necessary to reset the encoder has been trasmitted, obtain the encoded sequence associated to the following sequence of uncoded bits

$$B[m] = 1011001001\cdots$$