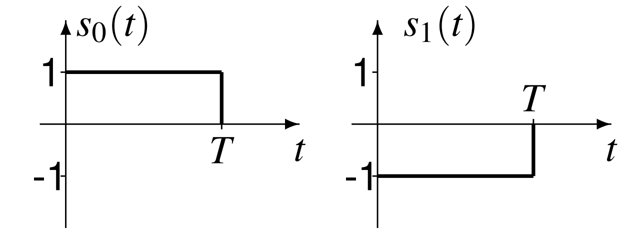
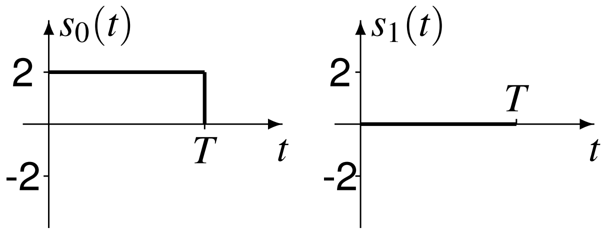
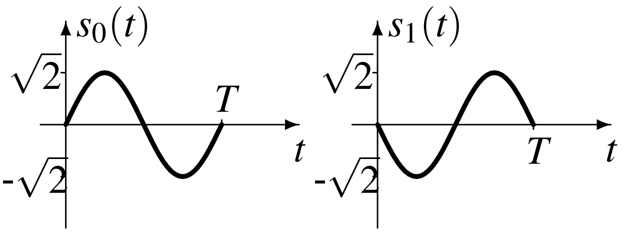
Modulador - Candidatos para $s_0(t)$, $s_1(t)$



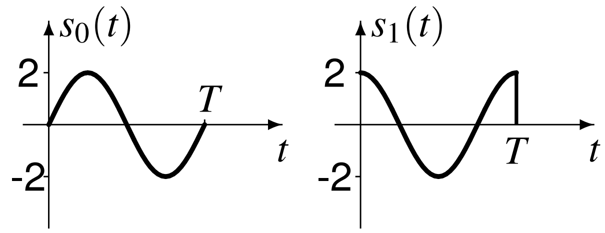
Conjunto 1



Conjunto 2



Conjunto 3



Conjunto 4

Distancias entre las señales

$$d(\mathbf{s}_i, \mathbf{s}_j) = ||\mathbf{s}_i - \mathbf{s}_j|| = \sqrt{\int_{-\infty}^{\infty} |s_i(t) - s_j(t)|^2 dt}$$

- Primer conjunto

$$d(\mathbf{s}_0, \mathbf{s}_1) = \sqrt{\int_0^T |1 - (-1)|^2 dt} = 2\sqrt{T}$$

- Segundo conjunto

$$d(\mathbf{s}_0, \mathbf{s}_1) = \sqrt{\int_0^T |2 - 0|^2 dt} = 2\sqrt{T}$$

Distancias entre las señales

- Tercer conjunto

$$d(\mathbf{s}_0,\mathbf{s}_1) = \sqrt{\int_0^T \left|\sqrt{2}\sin\left(\frac{2\pi t}{T}\right) - \left(-\sqrt{2}\sin\left(\frac{2\pi t}{T}\right)\right)\right|^2 dt}$$

$$= \sqrt{\int_0^T 8\sin^2\left(\frac{2\pi t}{T}\right) dt} = \sqrt{4\left[t - \frac{T}{2\pi}\sin\left(\frac{2\pi t}{T}\right)\cos\left(\frac{2\pi t}{T}\right)\right]_0^T} = 2\sqrt{T}$$

- Cuarto conjunto

$$d(\mathbf{s}_0,\mathbf{s}_1) = \sqrt{\int_0^T \left|2\sin\left(\frac{2\pi t}{T}\right) - \left(2\cos\left(\frac{2\pi t}{T}\right)\right)\right|^2 dt}$$

$$= \sqrt{\int_0^T 4 - 8\sin\left(\frac{2\pi t}{T}\right)\cos\left(\frac{2\pi t}{T}\right) dt} = 2\sqrt{T}$$

ya que

$$\int_0^T 8\sin\left(\frac{2\pi t}{T}\right)\cos\left(\frac{2\pi t}{T}\right) dt = \left[\frac{2T}{\pi}\sin^2\left(\frac{2\pi t}{T}\right)\right]_0^T = 0$$

Energía media por símbolo

$$E_s = E\left[\mathcal{E}\{s(t)\}\right] = E\left[\int_{-\infty}^{\infty} |s(t)|^2 dt\right]$$

$$= \sum_{i=0}^{M-1} P\left(s(t) = s_i(t)\right) \int_{-\infty}^{\infty} |s_i(t)|^2 dt$$

- Conjunto 1

$$E_s = \frac{1}{2}\int_0^T |1|^2 dt + \frac{1}{2}\int_0^T |-1|^2 dt = \frac{1}{2}T + \frac{1}{2}T = T$$

- Conjunto 2

$$E_s = \frac{1}{2}\int_0^T |2|^2 dt + \frac{1}{2}\int_0^T |0|^2 dt = \frac{1}{2}4T + \frac{1}{2}0 = 2T$$

Energía media por símbolo

$$\int_0^T \cos^2\left(\frac{2\pi t}{T}\right) dt = \frac{T}{2\pi}\left[\frac{\pi t}{T} + \frac{1}{2}\cos\left(\frac{2\pi t}{T}\right)\sin\left(\frac{2\pi t}{T}\right)\right]_0^T = \frac{T}{2}$$

$$\int_0^T \sin^2\left(\frac{2\pi t}{T}\right) dt = \frac{T}{2\pi}\left[\frac{\pi t}{T} - \frac{1}{2}\cos\left(\frac{2\pi t}{T}\right)\sin\left(\frac{2\pi t}{T}\right)\right]_0^T = \frac{T}{2}$$

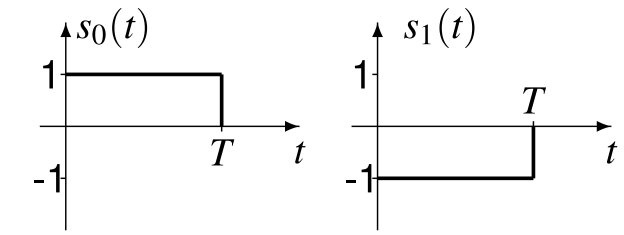
- Conjunto 3

$$E_s = \frac{1}{2}\cdot(\sqrt{2})^2\frac{T}{2} + \frac{1}{2}\cdot(\sqrt{2})^2\frac{T}{2} = T$$

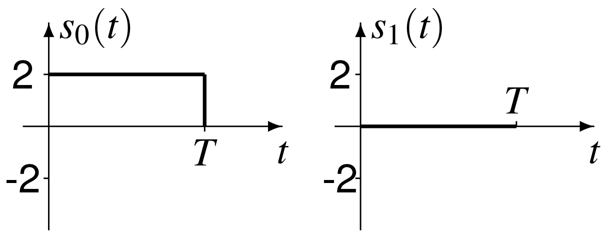
- Conjunto 4

$$E_s = \frac{1}{2}\cdot(2)^2\frac{T}{2} + \frac{1}{2}\cdot(2)^2\frac{T}{2} = 2T$$

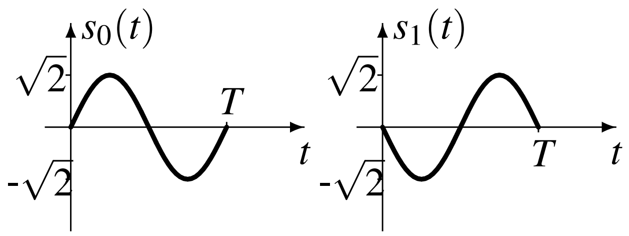
Modulador - Candidatos para $s_0(t)$, $s_1(t)$



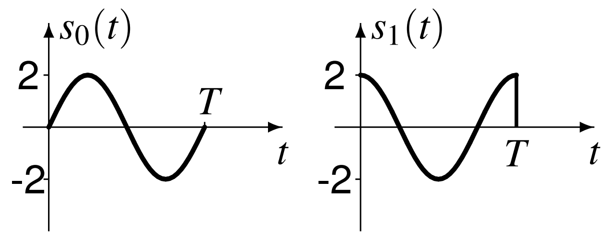
Conjunto 1



Conjunto 2

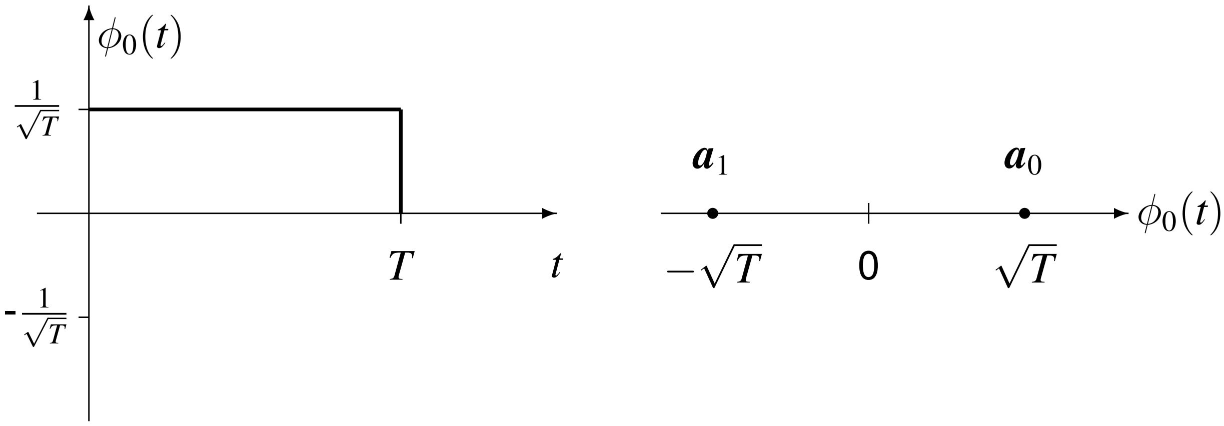


Conjunto 3



Conjunto 4

Base y constelación - Conjunto 1

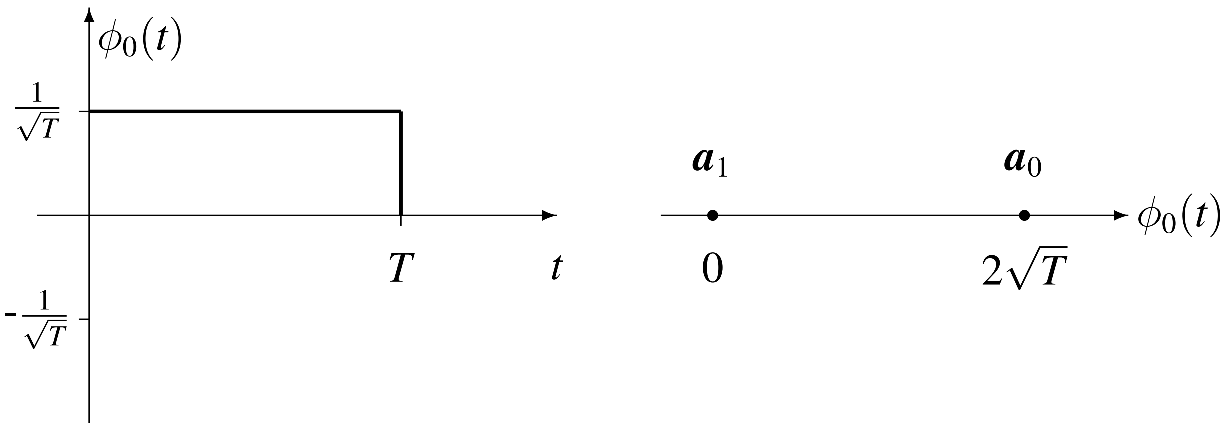


$$a_0 = [a_{0,0}] = +\sqrt{T}, \quad a_1 = [a_{1,0}] = -\sqrt{T}$$

$$s_0(t) = a_0 \cdot \phi_0(t)$$

$$s_1(t) = a_1 \cdot \phi_0(t)$$

Base y constelación - Conjunto 2

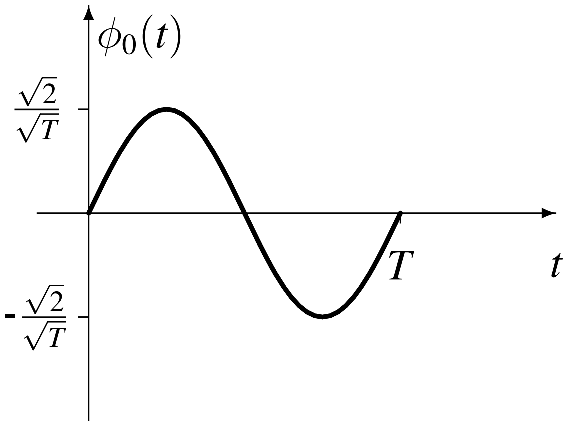


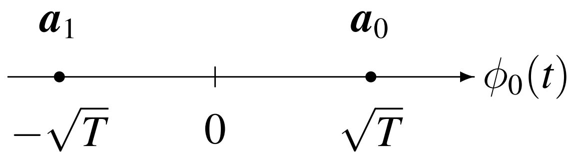
$$a_0 = [a_{0,0}] = +2\sqrt{T}, \quad a_1 = [a_{1,0}] = 0$$

$$s_0(t) = a_0 \cdot \phi_0(t)$$

$$s_1(t) = a_1 \cdot \phi_0(t)$$

Base y constelación - Conjunto 3



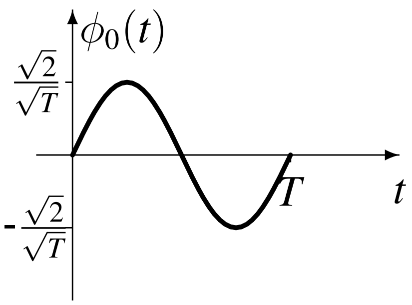


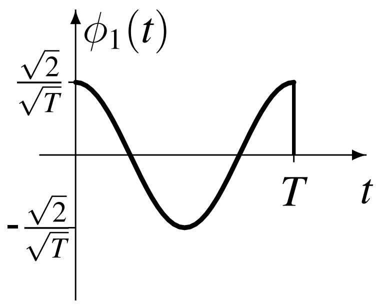
$$\boldsymbol{a}_0 = [a_{0,0}] = +\sqrt{T}, \quad \boldsymbol{a}_1 = [a_{1,0}] = -\sqrt{T}$$

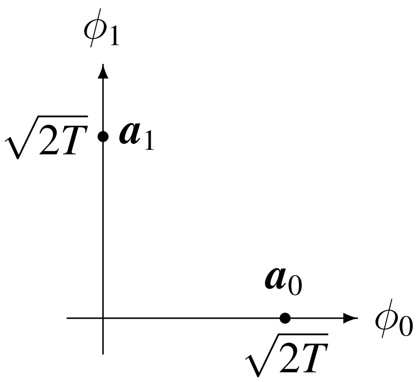
$$s_0(t) = \boldsymbol{a}_0 \cdot \phi_0(t)$$

$$s_1(t) = \boldsymbol{a}_1 \cdot \phi_0(t)$$

Base y constelación - Conjunto 4







$$\boldsymbol{a}_0 = \begin{bmatrix} a_{0,0} \\ a_{0,1} \end{bmatrix} = \begin{bmatrix} \sqrt{2T} \\ 0 \end{bmatrix}, \quad \boldsymbol{a}_1 = \begin{bmatrix} a_{1,0} \\ a_{1,1} \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{2T} \end{bmatrix}$$

$$s_0(t) = a_{0,0} \cdot \phi_0(t) + a_{0,1} \cdot \phi_1(t)$$

$$s_1(t) = a_{1,0} \cdot \phi_0(t) + a_{1,1} \cdot \phi_1(t)$$

Distancias entre las señales - Constelación

$$d(\boldsymbol{a}_i, \boldsymbol{a}_k) = ||\boldsymbol{a}_i - \boldsymbol{a}_j|| = \sqrt{\sum_{j=0}^{N-1} (a_{i,j} - a_{k,j})^2}$$

- Primer conjunto

$$d(\boldsymbol{a}_0, \boldsymbol{a}_1) = 2\sqrt{T}$$

- Segundo conjunto

$$d(\boldsymbol{a}_0, \boldsymbol{a}_1) = 2\sqrt{T}$$

- Tercer conjunto

$$d(\boldsymbol{a}_0, \boldsymbol{a}_1) = 2\sqrt{T}$$

- Cuarto conjunto

$$d(\boldsymbol{a}_0, \boldsymbol{a}_1) = 2\sqrt{T}$$

Energía media por símbolo - Constelación

$$E_s = E\left[\mathcal{E}\{s(t)\}\right] = \sum_{i=0}^{M-1} p_A\left(\boldsymbol{a}_i\right) \cdot \mathcal{E}\{\boldsymbol{a}_i\}$$

$$= \sum_{i=0}^{M-1} p_A\left(\boldsymbol{a}_i\right) \cdot \sum_{j=0}^{N-1} (a_{i,j})^2$$

- Conjunto 1 (símbolos equiprobables)

$$E_s = \frac{1}{2} \cdot \left(+\sqrt{T}\right)^2 + \frac{1}{2} \cdot \left(-\sqrt{T}\right)^2 = \frac{1}{2} \cdot T + \frac{1}{2} \cdot T = T$$

- Conjunto 2 (símbolos equiprobables)

$$E_s = \frac{1}{2} \cdot \left(+2\sqrt{T}\right)^2 + \frac{1}{2} \cdot (0)^2 = \frac{1}{2} \cdot 4T + \frac{1}{2} \cdot 0 = 2T$$

Energía media por símbolo - Constelación (II)

- Conjunto 3 (símbolos equiprobables)

$$E_s = \frac{1}{2} \cdot \left(+\sqrt{T}\right)^2 + \frac{1}{2} \cdot \left(-\sqrt{T}\right)^2 = \frac{1}{2} \cdot T + \frac{1}{2} \cdot T = T$$

- Conjunto 3 (símbolos equiprobables)

$$\begin{aligned} E_s =& \frac{1}{2} \cdot \left[\left(\sqrt{2T}\right)^2 + (0)^2\right] + \frac{1}{2} \cdot \left[(0)^2 + \left(\sqrt{2T}\right)^2\right] \\ =& \frac{1}{2} \cdot 2T + \frac{1}{2} \cdot 2T = 2T \end{aligned}$$

- Mínima energía para unas distancias entre símbolos dadas

$$E\left[\boldsymbol{a}_i\right] = \boldsymbol{0} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$