Homework1

Due Date: May 21st

You can either email your answer to `fernando@tsc.uc3m.es` before the afternoon lecture or hand it in during the afternoon lecture.

1. In the application to spectrometry prove

$$E_s = E_1 - E_2 \iff R_x(k) + R_y(k) = 2L\delta_{k,0}$$

   Note: You only need to show this for the Golay complimentary pairs in Slide 6 of Lecture 1.

   Hint: For the energy of the desired signal S, we only need to look at $k = 0$. For the background we need to look at $k \neq 0$. Remember that the top grating allows the signal and the background to cross and the lower grating only allows for the background and it cancels the signal. So the difference must be the desired signal. You need to relate the number of blocks that cross from the left to the right-hand side with the correlation between x and x, $\bar{x}$ and $\bar{x}$, y and $\bar{y}$, and $\bar{y}$ and y. It is also useful to look at the relation between the correlation of y and $\bar{y}$ with respect to the correlation of y and y, to obtain the final relation.

2. Calculate the Weyl transform of the vector φ given by

$$\varphi = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \\ \mathrm{i} \\ \mathrm{i} \\ -\mathrm{i} \\ \mathrm{i} \end{bmatrix}$$

Hint: First, you need to compute $S = \varphi\varphi^\dagger$. Second, the nonzero values of the Weyl transform are given by $D(a, aP)$, where $P = \begin{pmatrix} 110 \\ 101 \\ 011 \end{pmatrix}$. There are at most 8 of them.