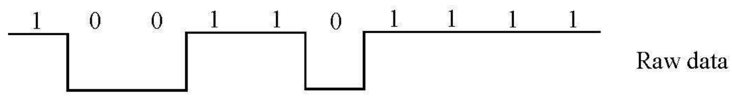
ELE 539B: Coding Theory

Homework 4: Spectra of Line Codes

No.1. Calculate a closed form expression for the spectrum of the line code described below under the assumption that the input data bits a_k are independent and equiprobable.



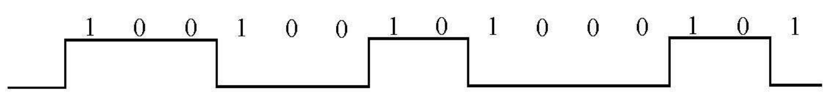
The Miller code is a rate 1/2 code. It is just a rule for inserting a binary digit between data bits. The Miller codeword corresponding to raw data (a_k) is $(\overline{a}_{k-1}\overline{a}_k, a_k)$. Thus

$$(0,0) \to (0,1,0),\ (1,0) \to (1,0,0),\ (0,1) \to (0,0,1),\ \text{and } (1,1) \to (1,0,1).$$

The codeword corresponding to the data sequence given above is

1 0 0 1 0 0 1 0 1 0 0 0 1 0 1

The modification rule is that the symbol 1 determines a change in level and that the symbol 0 determines no change in level. The Miller codeword given above is written as



The two levels are ± 1 so that the Miller codeword (y_{2k}, y_{2k+1}) is the sequence

$$1,\ 1,\ 1,\ -1,\ -1,\ -1,\ 1,\ 1,\ -1,\ -1,\ -1,\ -1,\ 1,\ 1,\ -1.$$

The line code (z_ℓ) is obtained by passing the Miller codeword (y_ℓ) through a 1-D filter. Thus

$$z_{2k} = y_{2k} - y_{2k-1} \ \text{ and } \ z_{2k+1} = y_{2k+1} - y_{2k}.$$

(A) There are 8 channel states corresponding to the triples $(a_{k-2}, a_{k-1}; y_{2k-3})$, and from any given state there are two equiprobable transitions corresponding to the different values of a_k. This transition is labeled with the codeword (z_{2k}, z_{2k+1}). Complete the state transition matrix given below.

	(00;1)	(00;-1)	(01;1)	(01;-1)	(10;1)	(10;-1)	(11;1)	(11;-1)
(00;1)		(2,0)		(0,2)				
(00;-1)								
(01;1)								
(01;-1)							(0,-2)	
(10;1)								
(10;-1)		(2,0)						
(11;1)								
(11;-1)					(0,0)		(0,-2)	

Sample calculation: initial state $(00;1)$, $a_k = 1$. Here $y_{2k-3} = 1$, $a_{k-2} = 0$, $a_{k-1} = 0$, $a_k = 1$. The corresponding Miller codeword is 1001 which is written as

$$(y_j)_{j=2k-2}^{2k+1} = (-1, -1, -1, 1).$$

Now $(z_{2k}, z_{2k+1}) = (-1-(-1), 1-(-1)) = (0, 2)$ and the new state is $(0, 1; -1)$.

Hint: $(ab; d) \overset{(z_1,z_2)}{\longrightarrow} (bc; e)$ if and only if $(ab; \overline{d}) \overset{(-z_1,-z_2)}{\longrightarrow} (bc; -e)$.

(B) Calculate the 2×8 matrix e with entries the sum of the signal points that can be transmitted when entering state ℓ weighted by the probability of their appearance.

$e =$

(00;1)	(00;-1)	(01;1)	(01;-1)	(10;1)	(10;-1)	(11;1)	(11;-1)
-1/4	1/4						
0	0						

Sample calculation: The equilibrium probability distribution is uniform so the weighted sum of the signal points that can be transmitted on entering state $(00; -1)$ is $\frac{1}{8} \times \frac{1}{2} \times ((2,0) + (2,0)) = (1/4, 0)$.

Calculate the 8×2 matrix d with entries the average of the signal points that can be transmitted from a given state.

$$d = \left[\begin{array}{c|c} 1 & 1 \\ \hline -1 & -1 \\ \hline & \\ \hline & \\ \hline & \\ \hline & \\ \hline & \\ \hline & \end{array}\right] \begin{array}{l} (00;1) \\ (00;-1) \\ (01;1) \\ (01;-1) \\ (10;1) \\ (10;-1) \\ (11;1) \\ (11;-1) \end{array}$$

(C) Let P be the matrix of the state transition probabilities. Calculate ed, ePd, eP^2d, eP^3d, and eP^4d. Prove that $eP^kd = -eP^{k-4}d/4$ for $k \geq 4$.

(D) Let $z = e^{i\theta}$. The spectrum $S(z)$ is given by

$$S(z) = R_0 + f(z) + f(z^{-1})$$

where R_0 is the average transmitted signal power and

$$f(z) = \sum_{k \geq 1} R_k z^k.$$

Prove that $R_0 = 3/2$ and that

$$f(z) = \frac{-12z^2 - 8z^3 + 2z^4 + 8z^5 + 4z^6 - 4z^7 - 5z^8}{4(4 + z^8)}.$$

(E) Graph the spectrum $S(z)$.