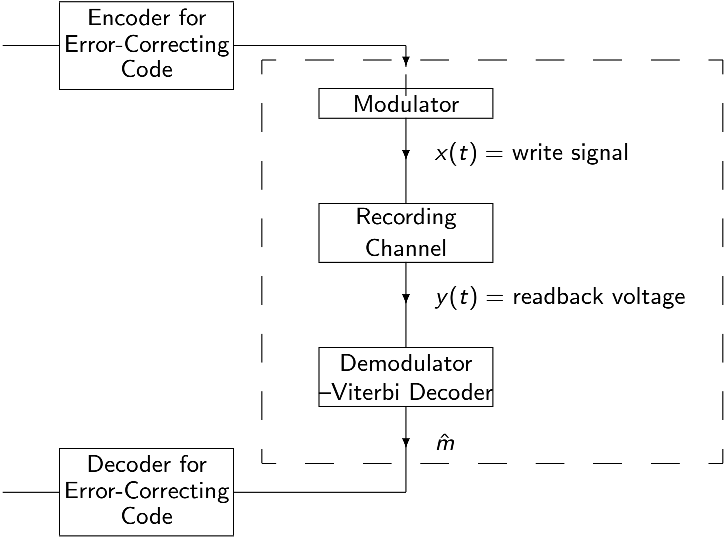
ELE 539B: Coding Theory

Lecture 4

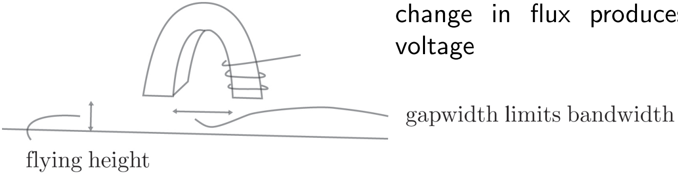
Magnetic Recording Channel – Mathematical Model
Peak Detection and Run Length Limited Codes
Constrained Coding
Partial Response and Gains over Threshold Detection

Reference: K.A. Schouhamer Immink, Codes for Mass Data Storage Systems, Shannon Foundation Publishers, The Netherlands.

Magnetic Recording Channel



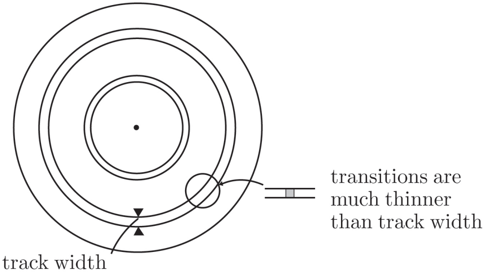
The Recording Channel



change in flux produces output voltage

Velocity of head is different on inner and out tracks

- need to equalize the different responses to a uniform channel.

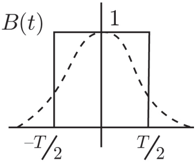


Noise

- additive white Gaussian noise due to electronics
- additive colored Gaussian noise due to noisy media filtered through read head
- signal dependent noise
- timing jitter
- dropouts: greatly reduced amplitude from imperfections in media or head clogging
- sidereading: signal from neighboring track or from older signal between tracks
- noncomplete erasure: old signal on track is not total erased when new signal is written

Value of Coding: Higher Areal Densities
Greater immunity to variance in quality of components

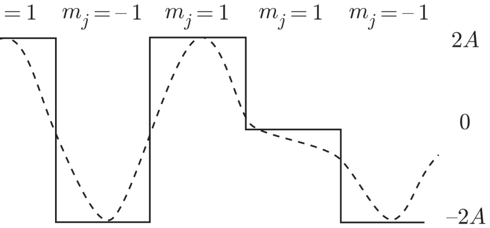
Mathematical Model



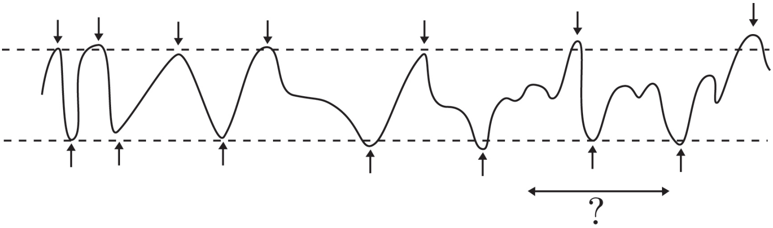
$p(t)$: response to isolated transition
T: minimum allowable spacing between transitions

(start by taking $T \approx$ width of $p(t)$)

$$y(t) = \sum A(m_j - m_{j-1}) p(t\text{-}jT) + z(t)$$



Peak Detection



Transitions at input are detected by finding peaks at the output.

Successful operation of this scheme depends on the minimum spacing between transitions. If two transitions are too close the peaks are reduced in amplitude and shifted.

Peak Detection vs. Sequence Estimation: Peak detection looks at a signal with respect to itself not with respect to the universe of possible signal sequences.

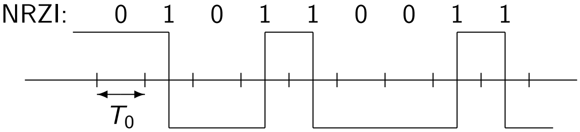
Run Length Limited Codes

NRZI data: 0 – no transition; 1 – transition

(d, k) constrained binary sequences

$$
\begin{aligned}
d &= \text{minimum number of 0's between adjacent 1's} \\
k &= \text{maximum number of 0's between adjacent 1's}
\end{aligned}
$$

$$
\begin{aligned}
T_{\text{min}} &= (d+1)\,T_0 \quad &\text{– maintains separation between transitions} \\
T_{\text{max}} &= (k+1)\,T_0 \quad &\text{– aids timing recovery}
\end{aligned}
$$



- rate loss from converting arbitrary binary sequences to (d, k) sequences
- rate gain from spacing data bits more finely than transitions

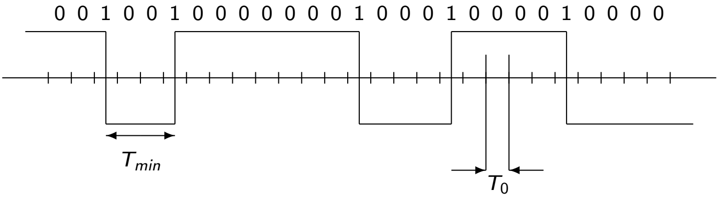
Comparison of Linear Information Densities

Variable Length Fixed Rate (2,7) Code: Rate $\frac{1}{2}$ code found in IBM 3370, 3375 and 3380 disk drives.

Data	Message
10	0100
11	1000
000	000100
010	100100

Data	Message
011	001000
0010	00100100
0011	00001000

(2,7) NRZI: Factor of 3 rate gain: $-T_{min} = 3T_0$

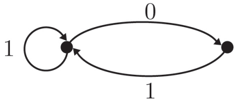


Value: 50% gain in a real density over uncoded NRZI

Finite State Machines

How to map binary data onto constrained sequences in a way that allows easy encoding and decoding.

Example: $(d, k) = (0, 1)$



Capacity C: the number of paths of length N through this graph grows like 2^{CN}

start with the state transition matrix: $A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$

find the largest real eigenvalue λ:

$$\text{characteristic polynomial is } x^2 - x - 1 : \lambda = (1 + \sqrt{5})/2$$

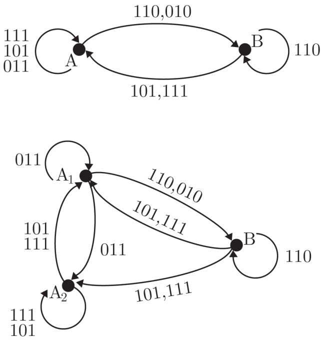
capacity $C = \log_2 \lambda = 0.6942$

Reference: B.H. Marcus, P.H. Siegel, and J.K. Wolf, Finite-state modulation codes for data storage, JSAC (10), 1992.

Constrained Coding

Theorem 4.1 Let S be a constrained system with Shannon capacity C and let p, q be positive integers satisfying $p/q \leq C$. Then there exists a finite state encoder with a state dependent decoder that encodes binary data into the constraint S at the constant rate $p : q$

Example: $(d, k) = (0, 1)$ with $2/3 \leq C = 0.6942$



This graph is derived from paths of length 3 in the original diagram

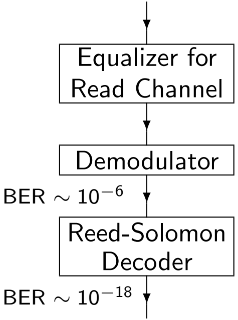
Problem: Only 3 edges leaving state B

The sequences generated by these two graphs are exactly the same but now there are 4 edges out of each state.

Systematic Procedure: state splitting (Perron-Frobenius linear algebra).

Partial Response

Forget about the d constraint and shrink the interval between transitions. Neighboring signals on the disk will interfere at the output. Use Viterbi detection to resolve this intersymbol interference.



$\longleftarrow$ Converts the recording channel to a tractable response

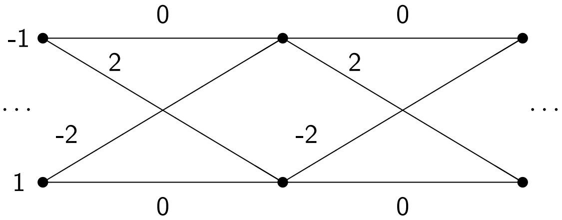
$$\text{PRML} : 1 - D^2$$

$$\text{EPR4} : (1 - D)(1 + D)^2$$

Note: $1 - D^2$ decouples into two interleaved $1 - D$ channels allowing two parallel decoders operating at half rate.

Gain over Threshold Detection

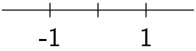
Viterbi Detection: $1 - D$ channel – $d^2 = 8$



State: last message bit

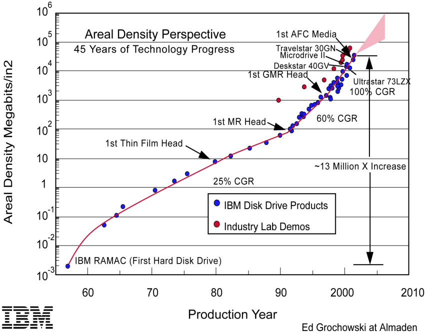
data is encoded as a path through the trellis and the decoder uses the Viterbi algorithm to estimate this path. Every interval the decoder calculates and stores the most likely path terminating in a given state.

Threshold Detection: $d^2 = 4$



Gain: $10 \log_{10} \frac{8}{4} = 3dB$ – though this must be discounted by the cost in SNR of equalizing the channel to $1 - D^2$

Communications Technology in Magnetic Recording



Source:
`http://www.storage.ibm.com/technolo/grochows/g02.htm`

Recording Channels are Communications Channels

- 250M hard disk drives sold in 2003
- Hard drives in MP3 players and set top boxes (TiVo)
- 60% density increase per year

Role of Communications

With each advance in head and media comes a loss in SNR – this is compensate for by communications technology

Standard tools of communications in every hard drive

- coding, signal processing central to low cost and reliability

Used for density increases and manufacturing tolerances

- small improvements in coding efficiency result in dramatic improvements in manufacturing yields and system margin