Radar Signal Processing

OFDM Radar

Golay Pairs and Frequency Multiplexing

- OFDM is the future technology in communications.
- Fundamental challenge in enabling OFDM radar is that multiplexing in frequency leads to range-dependent phase terms which destroy a lot of the intended benefit.
- This is particularly true for complementary waveforms (which are orthogonal at all lags).
- Employ non-linear signal processing to enable complementary behavior to be achieved by codes which are separated in frequency.

Golay Pairs and Frequency Multiplexing

- Let (p_1, p_2) be a Golay pair corresponding to coded waveforms

$$s_i(t) = \sum_{\ell=0}^{L-1} p_i(\ell) u(t - \ell \Delta T), \quad i = 1, 2.$$

- Frequency multiplexing: Transmit $s_1(t)$ and $s_2(t)$ simultaneously via frequency separation

$$\tilde{s}(t) = (s_1(t) + s_2(t) e^{j\omega_b t}) e^{j\omega_c t}$$

- Received signal:

$$\begin{aligned} \tilde{y}(t) &= a\tilde{s}(t-d) \\ &= a s_1(t-d) e^{j\omega_c (t-d)} + a s_2(t-d) e^{j(\omega_c + \omega_b)(t-d)} \end{aligned}$$

Golay Pairs and Frequency Multiplexing

- Down-modulate and lowpass filter:

$$
\begin{aligned}
y_1(t) &= a s_1(t-d) e^{-j\omega_c d} \\
y_2(t) &= a s_2(t-d) e^{-j(\omega_c+\omega_b)d}
\end{aligned}
$$

- Cross-correlate with transmit signals and sum:

$$
\begin{aligned}
R_{s_1 y_1}(\tau) &= a e^{-j\omega_c d} R_{s_1}(\tau - d) \\
R_{s_2 y_2}(\tau) &= a e^{-j(\omega_c+\omega_b)d} R_{s_2}(\tau - d)
\end{aligned}
$$

$$
R_{s_1 y_1}(\tau) + R_{s_2 y_2}(\tau) = a e^{-j\omega_c d}\left(R_{s_1}(\tau - d) + R_{s_2}(\tau - d) e^{-j\omega_b d}\right)
$$

- Complementary property is destroyed because of the range dependent phase shift.

Equal and Opposite Offsets

How to solve this problem? Equal and opposite offsets!

- If $s_2(t)$ had been frequency modulated by ω_b in the negative direction then the down-modulated signal would be

$$\tilde{y}_{2b}(t) = as_2(t-d)e^{-j(\omega_c-\omega_b)\tau}$$

Then,

$$R_{s_2y_2}(\tau) \times R_{s_2y_{2b}}(\tau) = a^2e^{-j2\omega_c d}R^2_{s_2}(\tau-d)$$

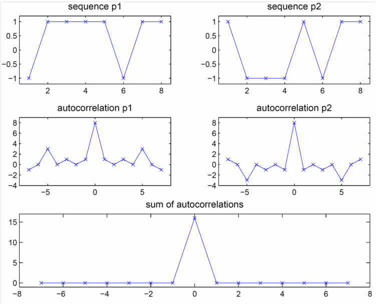
- Offset phase term cancels out.
- Gives square of the autocorrelation. Recovers autocorrelation up to a sign.

Modified Golay Pairs

- Design a pair of sequences (p, q) such that

$$R_p^2(k) + R_q^2(k) = \alpha\delta(k)$$

- At least one of the squared autocorrelations must be negative at some values of k.
- Possible only if the sequence has imaginary components.
- Autocorrelation of Golay Pairs:



$$R_{p_1}(k) = -R_{p_2}(k), \quad \forall k \neq 0$$

$$R_{p_1}^2(k) = R_{p_2}^2(k), \quad \forall k \neq 0$$

$$R_{p_1}(2k) = R_{p_2}(2k) = 0, \quad \forall k \neq 0$$

$$R_{p_1}(k) + R_{p_2}(k) = 2L\delta(k)$$

Modified Golay Pairs

- Let $p_1(\ell)$ and $p_2(\ell)$ be a Golay pair with

$$R_{p_1}(2k) = R_{p_2}(2k) = 0, \quad \forall k.$$

- Define

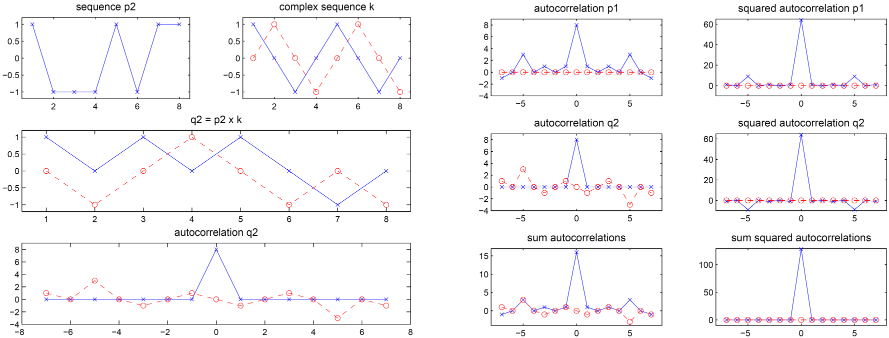
$$q_2(\ell) = p_2(\ell) e^{j\frac{\pi}{2}\ell} \longrightarrow R_{q_2}(k) = R_{p_2}(k) e^{j\frac{\pi}{2}k}$$

- *Modified Golay pair* (p_1, q_2)*:*

$$R^2_{q_2}(k) = R^2_{p_2}(k) e^{j\pi k} = \begin{cases} -R^2_{p_1}(k) & k \text{ odd} \\ 0 & k \neq 0 \text{ even} \\ R^2_{p_1}(k) & k = 0 \end{cases}$$

$$\longrightarrow R^2_{q_2}(k) + R^2_{p_1}(k) = 2L^2\delta(k)$$

Modified Golay Pairs



$$R^2_{q_2}(k) = R^2_{p_2}(k)e^{j\pi k} = \begin{cases} -R^2_{p_1}(k) & k \text{ odd} \\ 0 & k \neq 0 \text{ even} \\ R^2_{p_1}(k) & k = 0 \end{cases} \longrightarrow R^2_{q2}(k) + R^2_{p_1}(k) = 2L^2\delta(k)$$

Modified Golay Pairs for Radar

- Modified Golay pair (p_1, q_2) is used to modulate a pulse:

$$s_1(t) = \sum_{\ell=0}^{L-1} p_1(\ell) u(t - \ell \Delta T) \quad \text{and} \quad s_2(t) = \sum_{\ell=0}^{L-1} q_2(\ell) u(t - \ell \Delta T)$$

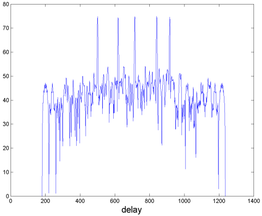
- First waveform is transmitted at carrier frequency. Second waveform is transmitted twice, offset equally above and below the carrier.
- Received signal:

$$\begin{aligned} y_1(t) &= a e^{-j\omega_c d} s_1(t-\tau) \\ y_{2a}(t) &= a e^{-j(\omega_c+\omega_b)d} s_2(t-\tau) \\ y_{2b}(t) &= a e^{-j(\omega_c-\omega_b)d} s_2(t-\tau) \end{aligned}$$

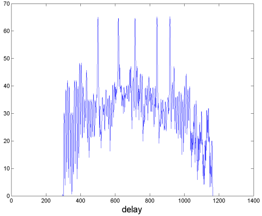
- Receiver signal processing:

$$\Gamma(\tau) = R^2_{s_1 y_1}(\tau) + R_{s_2 y_{2a}}(\tau) \times R_{s_2 y_{2b}}(\tau)$$

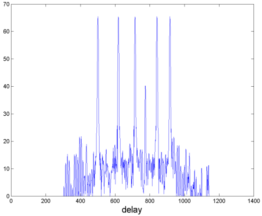
Varying the Offset Frequency



Frank pulse train



Modified Golay pulse train



Modified Golay pulse train

after varying offset

- A weak target in presence of five strong scatterers
- Frank and modified Golay pulse trains (left and middle) both suppress the sidelobes, but not enough.
- Changing the offset frequency (right) brings out the target.

References

1. S. J. Searle, S. D. Howard, and W. Moran, "On formation of composite ambiguity functions with frequency separated Golay coded pulses," *IEEE Trans. Aerosp. Electron. Syst.,* to appear 2009.
2. R. Calderbank, S. D. Hoawrd, and W. Moran, "Waveform diversity in radar signal processing: A focus on the use and control of degrees of freedom," *IEEE Signal Processing Magazine,* vol. 26, no. 1, pp. 32-41, Jan. 2009.
3. S. J. Searle and S. D. Howard, "A novel nonlinear technique for sidelobe suppression in radar," *Proc. International Conference on Radar Systems,* Edinburgh, UK, Oct. 2007.
4. S. Searle and S. Howard, "A novel polyphase code for sidelobe suppression," *Proc. Int. Waveform Diversity and Design Conf.,* Pisa, Italy, Jun. 4-8, 2007.