with power-spectral density $\frac{N_0}{2}$.

1. Find an appropriate orthonormal basis for the representation of the signals.
2. In a block diagram, give the precise specifications of the optimal receiver using matched filters. Label the block diagram carefully.
3. Find the error probability of the optimal receiver.
4. Show that the optimal receiver can be implemented by using just *one* filter [see block diagram shown in Figure P-7.33(b)]. What are the characteristics of the matched filter and the sampler and decision device?
5. Now assume the channel is not ideal, but has an impulse response of $c(t) = \delta(t) + \frac{1}{2}\delta(t - \frac{T}{2})$. Using the same matched filter you used in the previous part, design an optimal receiver.
6. Assuming that the channel impulse response is $c(t) = \delta(t) + a\delta(t - \frac{T}{2})$, where a is a random variable uniformly distributed on [0, 1], and using the same matched filter, design the optimal receiver.

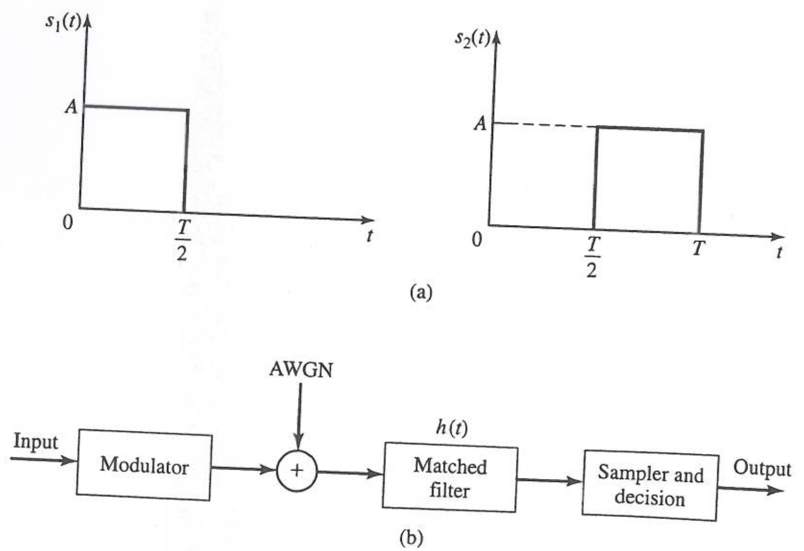

Figure P-7.33

7.34 Suppose that binary PSK is used for transmitting information over an AWGN with power-spectral density of $N_0/2 = 10^{-10}$ W/Hz. The transmitted signal energy is $\mathcal{E}_b = A^2T/2$, where T is the bit interval and A is the signal amplitude. Determine the signal amplitude required to achieve an error probability of 10^{-6}, if the data rate is (a) 10 kbps, (b) 100 kbs, (c) 1 Mbps

7.35 Consider the signal

$$u(t) = \begin{cases} \frac{A}{T}t\cos 2\pi f_c t, & 0 \le t \le T \\ 0, & \text{otherwise} \end{cases}$$

1. Determine the impulse response of the matched filter for the signal.
2. Determine the output of the matched filter at $t = T$.
3. Suppose the signal $u(t)$ is passed through a correlator which correlates the input $u(t)$ with $u(t)$. Determine the value of the correlator output at $t = T$. Compare your result with that in part (b).

7.36 A carrier component is transmitted on the quadrature carrier in a communication system that transmits information via binary PSK. Hence, the received signal has the form

$$v(t) = \pm\sqrt{2P_s}\cos(2\pi f_c t + \phi) + \sqrt{2P_c}\sin(2\pi f_c t + \phi) + n(t)$$

where ϕ is the carrier phase and $n(t)$ is AWGN. The unmodulated carrier component is used as a pilot signal at the receiver to estimate the carrier phase.

1. Sketch a block diagram of the receiver, including the carrier-phase estimator.
2. Illustrate mathematically the operations involved in the estimation of the carrier-phase ϕ.
3. Express the probability of error for the detection of the binary PSK signal as a function of the total transmitted power $P_T = P_s + P_c$. What is the loss in performance due to the allocation of a portion of the transmitted power to the pilot signal? Evaluate the loss for $P_c/P_T = 0.1$.

7.37 In the demodulation of a binary PSK signal received in white Gaussian noise, a phase-locked loop is used to estimate the carrier-phase ϕ.

1. Determine the effect of a phase error $\phi - \hat{\phi}$ on the probability of error.
2. What is the loss in SNR if the phase error $\phi - \hat{\phi} = 45°$?

7.38 Suppose that the loop filter [see Equation (5.2.4)] for a PLL has the transfer function

$$G(s) = \frac{1}{s + \sqrt{2}}$$

1. Determine the closed-loop transfer function $H(s)$ and indicate if the loop is stable.
2. Determine the damping factor and the natural frequency of the loop.

7.39 Consider the PLL for estimating the carrier phase of a signal in which the loop filter is specified as

K