7.27 Consider an M-ary digital communication system where $M = 2^N$, and N is the dimension of the signal space. Suppose that the M signal vectors lie on the vertices of a hypercube that is centered at the origin, as illustrated in Figure 7.29. Determine the average probability of a symbol error as a function of $\mathcal{E}_s/N_0$ where $\mathcal{E}_s$ is the energy/symbol, $N_0/2$ is the power-spectral density of the AWGN, and all signal points are equally probable.

7.28 Consider the signal waveform

$$s(t) = \sum_{k=1}^{n} c_i p(t - nT_c)$$

where $p(t)$ is a rectangular pulse of unit amplitude and duration T_c. The $\{c_i\}$ may be viewed as a code vector $\mathbf{c} = [c_1, c_2, \ldots, c_n]$, where the elements $c_i = \pm 1$. Show that the filter matched to the waveform $s(t)$ may be realized as a cascade of a filter matched to $p(t)$ followed by a discrete-time filter matched to the vector $\mathbf{c}$. Determine the value of the output of the matched filter at the sampling instant $t = nT_c$.

7.29 A speech signal is sampled at a rate of 8 kHz, logarithmically compressed and encoded into a PCM format using 8 bits/sample. The PCM data is transmitted through an AWGN baseband channel via M-level PAM. Determine the bandwidth required for transmission when (a) $M = 4$, (b) $M = 8$, and (c) $M = 16$.

7.30 Two equiprobable messages are transmitted via an additive white Gaussian noise channel with noise power-spectral density of $\frac{N_0}{2} = 1$. The messages are transmitted by the following two signals

$$s_1(t) = \begin{cases} 1 & 0 \le t \le 1 \\ 0, & \text{otherwise} \end{cases}$$

and $s_2(t) = s_1(t-1)$. It is intended to implement the receiver using a correlation type structure, but due to imperfections in the design of the correlators, the structure shown in Figure P-7.30 has been implemented. The imperfection appears in the integrator in the upper branch where instead of $\int_0^1$ we have $\int_0^{1.5}$. The decision

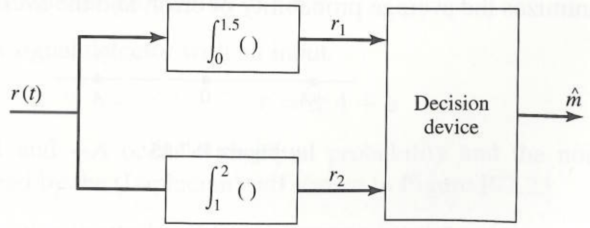

Figure P-7.30

device, therefore, observes r_1 and r_2 and based on this observation has to decide which message was transmitted. What decision rule should be adopted by the decision device for an optimal decision?

7.31 A Hadamard matrix is defined as a matrix whose elements are ± 1 and its row vectors are pairwise orthogonal. In the case when n is a power of 2, an $n \times n$ Hadamard matrix is constructed by means of the recursion

$$\mathbf{H}_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad \mathbf{H}_{2n} = \begin{bmatrix} \mathbf{H}_n & \mathbf{H}_n \\ \mathbf{H}_n & -\mathbf{H}_n \end{bmatrix}$$

1. Let $\mathbf{c}_i$ denote the ith row of an $n \times n$ Hadamard matrix as defined above. Show that the waveforms constructed as

$$s_i(t) = \sum_{k=1}^{n} c_{ik} p(t - kT_c), \quad i = 1, 2, \ldots, n$$

are orthogonal, where $p(t)$ is an arbitrary pulse confined to the time interval $0 \le t \le T_c$.

2. Show that the matched filters (or crosscorrelators) for the n waveforms $\{s_i(t)\}$ can be realized by a single filter (or correlator) matched to the pulse $p(t)$ followed by a set of n crosscorrelators using the code words $\{\mathbf{c}_i\}$.

7.32 The discrete sequence

$$r_k = \sqrt{\mathcal{E}_c} c_k + n_k, \quad k = 1, 2, \ldots, n$$

represents the output sequence of samples from a demodulator, where $c_k = \pm 1$ are elements of one of two possible codewords, $\mathbf{c}_1 = [1, 1, \ldots, 1]$ and $\mathbf{c}_2 = [1, 1, \ldots, 1, -1, \ldots, -1]$. The codeword $\mathbf{c}_2$ has w elements which are $+1$ and $n - w$ elements which are -1, where w is some positive integer. The noise sequence $\{n_k\}$ is white Gaussian with variance σ^2.

1. What is the optimum maximum-likelihood detector for the two possible transmitted signals?
2. Determine the probability error as a function of the parameter $(\sigma^2, \mathcal{E}_b, w)$.
3. What is the value of w that minimizes the error probability?

7.33 A baseband digital communication system employs the signals shown in Figure P-7.33(a) for transmission of two equiprobable messages. It is assumed the communication problem studied here is a "one shot" communication problem, that is, the above messages are transmitted just once and no transmission takes place afterwards. The channel has no attenuation ($\alpha = 1$) and the noise is AWG