3. Determine the variance the noise of the output of the matched filter at $t = 3$.
4. Determine the probability of error as a function of A and N_0.

7.15 A matched filter has the frequency response

$$H(f) = \frac{1 - e^{-j2\pi fT}}{j2\pi f}$$

1. Determine the impulse response $h(t)$ corresponding to $H(f)$.
2. Determine the signal waveform to which the filter characteristic is matched.

7.16 Prove that when a sinc pulse $g_T(t)$ is passed through its matched filter, the output is the same sinc pulse.

7.17 The demodulation of the binary antipodal signals

$$s_1(t) = -s_2(t) = \begin{cases} \sqrt{\frac{\mathcal{E}_b}{T}}, & 0 \le 1 \le T \\ 0, & \text{otherwise} \end{cases}$$

can be accomplished by use of a single integrator, as shown in Figure P-7.17, which is sampled periodically at $t = kT$, $k = 0, \pm 1, \pm 2, \ldots$. The additive noise is zero-mean Gaussian with power-spectral density of $\frac{N_0}{2}$ W/Hz.

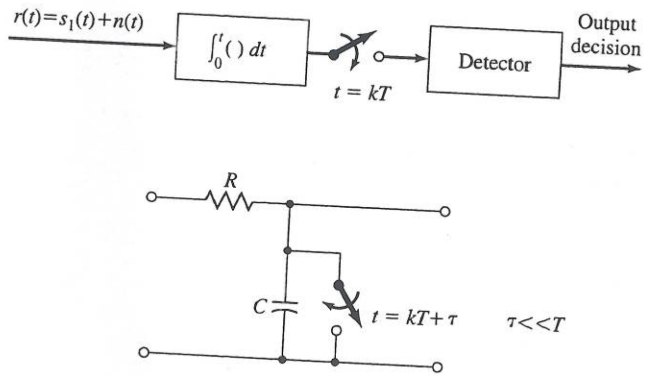

Figure P-7.17

1. Determine the output SNR of the demodulator at $t = T$.
2. If the ideal integrator is replaced by the RC filter shown in Figure P-7.17, determine the output SNR as a function of the time constant RC.
3. Determine the value of RC that maximizes the output SNR.

7.18 Sketch the impulse response of the filter matched to the pulses shown in Figure P-7.18. Also determine and sketch the outputs of each of the matched filters.

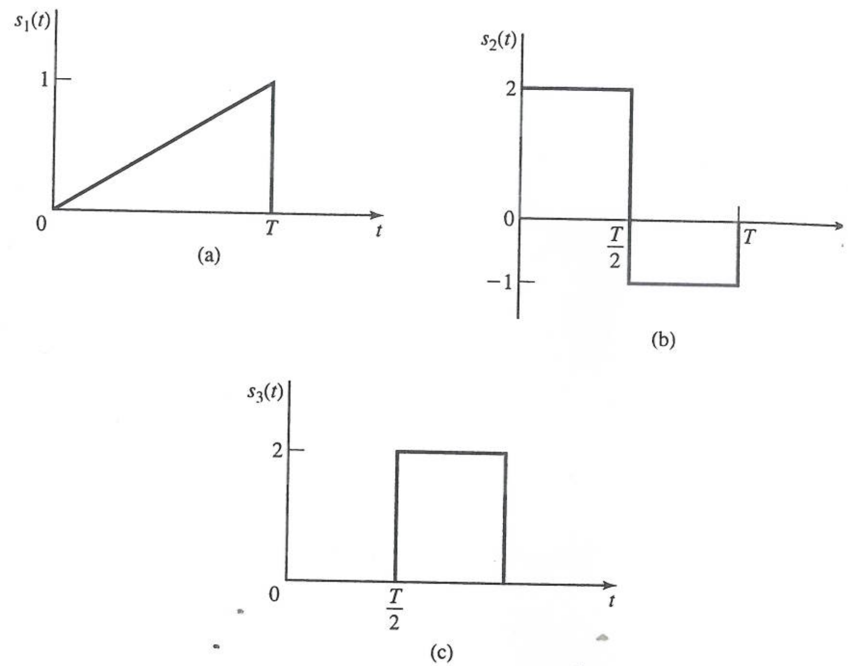

Figure P-7.18

7.19 Three messages m_1, m_2, and m_3 are to be transmitted over an AWGN cha with noise power-spectral density $\frac{N_0}{2}$. The messages are

$$s_1(t) = \begin{cases} 1 & 0 \le 1 \le T \\ 0, & \text{otherwise} \end{cases}$$

$$s_2(t) = -s_3(t) = \begin{cases} 1 & 0 \le t \le \frac{T}{2} \\ -1 & \frac{T}{2} \le 0 \le T \\ 0, & \text{otherwise} \end{cases}$$

1. What is the dimensionality of the signal space?
2. Find an appropriate basis for the signal space (Hint: You can find the b without using the Gram-Schmidt procedure).
3. Draw the signal constellation for this problem.
4. Derive and sketch the optimal decision regions R_1, R_2, and R_3.
5. Which of the three messages is more vulnerable to errors and why? In o words which of $P(\text{Error} \mid m_i \text{ transmitted})$, $i = 1, 2, 3$ is larger?