

A Rate-Splitting Approach to Fading Multiple-Access Channels with Imperfect Channel-State Information

Adriano Pastore*, Tobias Koch†, Javier R. Fonollosa*

* Universitat Politècnica de Catalunya, Signal Theory and Communications Department, 08034 Barcelona, Spain
Email: {adriano.pastore,javier.fonollosa}@upc.edu

† Universidad Carlos III, Signal Theory and Communications Department, 28911 Leganés, Spain
Email: koch@tsc.uc3m.es

Abstract—As shown by Médard, the capacity of fading channels with imperfect channel-state information (CSI) can be lower-bounded by assuming a Gaussian channel input and by treating the unknown portion of the channel multiplied by the channel input as independent worst-case (Gaussian) noise. Recently, we have demonstrated that this lower bound can be sharpened by a rate-splitting approach: by expressing the channel input as the sum of two independent Gaussian random variables (referred to as layers), say $X = X_1 + X_2$, and by applying Médard’s bounding technique to first lower-bound the capacity of the virtual channel from X_1 to the channel output Y (while treating X_2 as noise), and then lower-bound the capacity of the virtual channel from X_2 to Y (while assuming X_1 to be known), one obtains a lower bound that is strictly larger than Médard’s bound. This rate-splitting approach is reminiscent of an approach used by Rimoldi and Urbanke to achieve points on the capacity region of the Gaussian multiple-access channel (MAC). Here we blend these two rate-splitting approaches to derive a novel inner bound on the capacity region of the memoryless fading MAC with imperfect CSI. Generalizing the above rate-splitting approach to more than two layers, we show that, irrespective of how we assign powers to each layer, the supremum of all rate-splitting bounds is approached as the number of layers tends to infinity, and we derive an integral expression for this supremum. We further derive an expression for the vertices of the best inner bound, maximized over the number of layers and over all power assignments.

I. INTRODUCTION

Consider a discrete-time, memoryless, fading channel with imperfect channel-state information (CSI), whose time- k output ($k \in \mathbb{Z}$), conditioned on the channel input $X[k] = x \in \mathbb{C}$, is

$$Y[k] = (\hat{H}[k] + \tilde{H}[k])x + Z[k] \quad (1)$$

(with \mathbb{C} and \mathbb{Z} denoting the set of complex numbers and the set of integers, respectively). Here, the noise $\{Z[k]\}_{k \in \mathbb{Z}}$ is

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a sequence of independent and identically distributed (i.i.d.), zero-mean, circularly-symmetric, complex Gaussian random variables with variance σ^2 . The fading processes $\{\hat{H}[k]\}_{k \in \mathbb{Z}}$ and $\{\tilde{H}[k]\}_{k \in \mathbb{Z}}$ are both sequences of i.i.d. complex random variables (of arbitrary distribution), the former with mean μ and variance \hat{V} and the latter with mean zero and variance \tilde{V} . Assume that the processes $\{\hat{H}[k]\}_{k \in \mathbb{Z}}$, $\{\tilde{H}[k]\}_{k \in \mathbb{Z}}$, and $\{Z[k]\}_{k \in \mathbb{Z}}$ are independent of each other and of the input sequence $\{X[k]\}_{k \in \mathbb{Z}}$. Further assume that the receiver is cognizant of the realization of $\{\hat{H}[k]\}_{k \in \mathbb{Z}}$, but the transmitter is only cognizant of its distribution. Finally assume that both the transmitter and receiver are cognizant of the distributions of $\{\tilde{H}[k]\}_{k \in \mathbb{Z}}$ and $\{Z[k]\}_{k \in \mathbb{Z}}$ but not of their realizations.

The fading process $\{H[k]\}_{k \in \mathbb{Z}}$ can be viewed as an estimate of the channel fading coefficient

$$H[k] \triangleq \hat{H}[k] + \tilde{H}[k], \quad k \in \mathbb{Z} \quad (2)$$

and $\{\tilde{H}[k]\}_{k \in \mathbb{Z}}$ can be viewed as the channel estimation error.

The capacity of the above channel (1) under the average-power constraint P is given by [1]

$$C(P) = \sup I(X; Y | \hat{H}) \quad (3)$$

where the supremum is over all distributions of X satisfying $\mathbb{E}[|X|^2] \leq P$. Here and throughout the paper we omit the time indices k where they are immaterial. Since (3) is difficult to evaluate, it is common to assess $C(P)$ using upper and lower bounds. A well-known lower bound is due to Médard [2]:

$$C(P) \geq \mathbb{E} \left[\log \left(1 + \frac{|\hat{H}|^2 P}{\tilde{V} P + \sigma^2} \right) \right]. \quad (4)$$

It is derived by assuming a Gaussian channel input X and by treating the term $\tilde{H}X + Z$ as independent worst-case (Gaussian) noise.

In [3], it was demonstrated that (4) can be sharpened by a rate-splitting and successive decoding approach: writing the input $X = X_1 + X_2$ as a sum of two independent Gaussian random variables (referred to as *layers*) of respective powers P_1 and P_2 , using the chain rule

$$I(X; Y | \hat{H}) = I(X_1; Y | \hat{H}) + I(X_2; Y | \hat{H}, X_1) \quad (5)$$

and applying Médard's bound on each term, we obtain a lower bound that is strictly larger than (4) except in the trivial cases where $P_1 = 0$, $P_2 = 0$, or $\Pr(\tilde{H}\tilde{V} = 0) = 1$. This rate-splitting approach can be generalized to more than two layers. It was demonstrated that the supremum of all such rate-splitting bounds is approached as the number of layers tends to infinity and an integral expression of this supremum was presented [3, Theorem 4].

The above rate-splitting approach is reminiscent of a rate-splitting approach proposed by Rimoldi and Urbanke to achieve points on the capacity region of the Gaussian multiple-access channel [4]. For example, for the two-user Gaussian MAC, Rimoldi and Urbanke showed that any point in the capacity region can be achieved by splitting one user, say User 1, into two virtual users,² and by decoding first the codeword of the first virtual user while treating the codewords of the second virtual user and of User 2 as noise, by then decoding the codeword of User 2 upon subtracting the contribution of the first virtual user and treating the codeword of the second virtual user as noise, and by finally decoding the codeword of the second virtual user upon subtracting the contributions of the first virtual user and User 2.

In this paper, we blend the two rate-splitting approaches in [3] and [4] to derive a novel inner bound on the capacity region of the memoryless fading MAC with imperfect CSI. We show that, irrespective of how we assign powers to each layer, the supremum of all such rate-splitting bounds is approached as the number of layers tends to infinity, and we derive an integral expression for this supremum. We further derive an expression for the best inner bound, maximized over the number of layers and all power assignments.

II. CHANNEL MODEL AND CAPACITY REGION

We consider the multiple-access generalization of (1): the time- k output $Y[k]$, conditioned on the channel inputs $X_1[k] = x_1 \in \mathbb{C}$ and $X_2[k] = x_2 \in \mathbb{C}$ corresponding to User 1 and User 2, respectively, is

$$Y[k] = (\hat{H}_1[k] + \tilde{H}_1[k])x_1 + (\hat{H}_2[k] + \tilde{H}_2[k])x_2 + Z[k] \quad (6)$$

where $\{Z[k]\}_{k \in \mathbb{Z}}$ is as in Section I, and where, for each user $i = 1, 2$, the fading processes $\{\hat{H}_i[k]\}_{k \in \mathbb{Z}}$ and $\{\tilde{H}_i[k]\}_{k \in \mathbb{Z}}$ are sequences of i.i.d. complex random variables, the former with mean μ_i and variance \tilde{V}_i , and the latter with mean zero and variance \tilde{V}_i . We assume that the processes $\{\hat{H}_i[k]\}_{k \in \mathbb{Z}}$, $\{\tilde{H}_i[k]\}_{k \in \mathbb{Z}}$ ($i = 1, 2$) and $\{Z[k]\}_{k \in \mathbb{Z}}$ are independent of each other and of the input sequences $\{X_i[k]\}_{k \in \mathbb{Z}}$, $i = 1, 2$. As in Section I, we assume that both transmitter and receiver are cognizant of the distributions of $\{\hat{H}_i[k]\}_{k \in \mathbb{Z}}$, $\{\tilde{H}_i[k]\}_{k \in \mathbb{Z}}$ ($i = 1, 2$) and $\{Z[k]\}_{k \in \mathbb{Z}}$, and that the receiver is, in addition, cognizant of the realizations of $\{\tilde{H}_i[k]\}_{k \in \mathbb{Z}}$, $i = 1, 2$.

The capacity region of the above channel (6) under the power constraints P_1 and P_2 is given by the closure of the

²The virtual users correspond to the layers in [3].

convex hull of all rates (R_1, R_2) satisfying

$$R_1 \leq I(X_1; Y|X_2, \hat{\mathbf{H}}) \triangleq I_{1|2} \quad (7a)$$

$$R_2 \leq I(X_2; Y|X_1, \hat{\mathbf{H}}) \triangleq I_{2|1} \quad (7b)$$

$$R_1 + R_2 \leq I(X_1, X_2; Y|\hat{\mathbf{H}}) \triangleq I_{\Sigma} \quad (7c)$$

for some product distributions of (X_1, X_2) satisfying $\mathbb{E}[|X_1|^2] \leq P_1$ and $\mathbb{E}[|X_2|^2] \leq P_2$ [5].

In [2, Equations (69)–(71)], an inner bound on the capacity region was derived by assuming zero-mean real Gaussian channel inputs and by lower-bounding the mutual informations $I_{1|2}$, $I_{2|1}$ and I_{Σ} using worst-case noise bounds like (4). In the following, we will derive an improved capacity inner bound (for complex signalling) by evaluating (7a)–(7c) for zero-mean, circularly-symmetric, complex Gaussian channel inputs of respective powers P_1 and P_2 , and by using Médard's lower bound (4) together with the above presented rate-splitting approaches. Specifically, we follow the approach by Rimoldi and Urbanke [4] to characterize points (R_1, R_2) on the dominant face of (7a)–(7c), i.e., points satisfying

$$\begin{bmatrix} R_1 \\ R_2 \end{bmatrix} = (1 - \alpha) \begin{bmatrix} I_{\Sigma} - I_{2|1} \\ I_{2|1} \end{bmatrix} + \alpha \begin{bmatrix} I_{1|2} \\ I_{\Sigma} - I_{1|2} \end{bmatrix} \quad (8)$$

for some $0 \leq \alpha \leq 1$, by single-user constraints for each R_1 and R_2 . We then follow the rate-splitting approach presented in [3] to derive evaluable lower bounds on these single-user constraints.

To illustrate this approach, let us split User 1 into two virtual users, i.e., let $X_1 = X_{11} + X_{12}$, where X_{11} and X_{12} are independent, zero-mean, circularly symmetric, complex Gaussian random variables of respective powers $(1 - \beta)P_1$ and βP_1 . By performing successive decoding of X_{11} , X_2 and X_{12} (in this order), we can achieve the rates

$$R_{11} = I(X_{11}; Y|\hat{\mathbf{H}}) \quad (9a)$$

$$R_{12} = I(X_{12}; Y|\hat{\mathbf{H}}, X_{11}, X_2) \quad (9b)$$

$$R_2 = I(X_2; Y|\hat{\mathbf{H}}, X_{11}) \quad (9c)$$

giving rise to the single-user constraints

$$R_1 \leq I(X_{11}; Y|\hat{\mathbf{H}}) + I(X_{12}; Y|\hat{\mathbf{H}}, X_{11}, X_2) \quad (10a)$$

$$R_2 \leq I(X_2; Y|\hat{\mathbf{H}}, X_{11}). \quad (10b)$$

The mutual informations on the right-hand side (RHS) of (10a)–(10b) can then be lower-bounded following the rate-splitting approach presented in [3]. In this example, we first decode all layers of X_{11} , then all layers of X_2 , and finally all layers of X_{12} . By introducing more than two virtual users, we can construct different decoding orders that potentially give rise to sharper inner bounds.

III. POWER ALLOCATIONS AND INNER BOUNDS

The most general rate-splitting scheme on the two-user MAC can be represented as follows: the transmit signals of User $i = 1, 2$ are written as sums of independent, zero-mean, circularly-symmetric, complex Gaussian random variables $X_{i,\ell}$, $\ell = 1, \dots, L$ with respective powers $P_{i,\ell} \geq 0$

summing up to P_i , i.e.,

$$X_i = \sum_{\ell \in L} X_{i,\ell} \quad \text{and} \quad P_i = \sum_{\ell \in L} P_{i,\ell}. \quad (11)$$

The signals are decoded in an alternating decoding order

$$X_{1,1}, X_{2,1}, X_{1,2}, X_{2,2}, \dots, X_{1,L}, X_{2,L}. \quad (12)$$

This yields the rate pair

$$J_1 = \sum_{\ell=1}^L I(X_{1,\ell}; Y \mid \mathbf{X}_1^{\ell-1}, \mathbf{X}_2^{\ell-1}, \hat{\mathbf{H}}) \quad (13a)$$

$$J_2 = \sum_{\ell=1}^L I(X_{2,\ell}; Y \mid \mathbf{X}_1^\ell, \mathbf{X}_2^{\ell-1}, \hat{\mathbf{H}}) \quad (13b)$$

where \mathbf{X}_i^j stands for the collection $X_{i,1}, \dots, X_{i,j}$. Note that this decoding order incurs no loss in generality, since setting a power $P_{i,\ell}$ to zero effectively suppresses the decoding step. With the decoding order held fixed, any rate-splitting scheme is fully described by the power allocations $\{P_{i,\ell}\}$. However, we shall find it convenient to define power allocations via so-called *layering functions*.

Definition 1. A continuous surjective non-decreasing function $K_i: [0; 1] \rightarrow [0; 1]$ is called a *layering function* for user i . The set of layering functions is denoted as \mathcal{K} .

We shall define a rate-splitting scheme by the pair of layering functions $\mathbf{K} = (K_1, K_2) \in \mathcal{K}^2$ and the number of layers L . The corresponding power allocations can then be obtained by

$$P_{i,\ell} = P_i \left(K_i\left(\frac{\ell}{L}\right) - K_i\left(\frac{\ell-1}{L}\right) \right). \quad (14)$$

Note that \mathbf{K} does not depend on L .

A. Infinite-layer rate region

Upon applying Médard's bound on each summand on the RHS of (13a)–(13b), a given rate-splitting scheme (\mathbf{K}, L) yields an achievable-rate pair $\underline{\mathbf{J}}(\mathbf{K}, L) \triangleq (J_1(\mathbf{K}, L), J_2(\mathbf{K}, L))$. The following theorem shows that, for any \mathbf{K} , the supremum over all rate pairs is approached as L tends to infinity.

Theorem 1. For every pair of layering functions \mathbf{K} , the supremum of $J_i(\mathbf{K}, L)$ over the number of layers is given by the Lebesgue-Stieltjes integral

$$\sup_{L \in \mathbb{N}} J_i(\mathbf{K}, L) = \lim_{L \rightarrow \infty} J_i(\mathbf{K}, L) = \int_0^1 f_i(\zeta) dK_i(\zeta) \quad (15)$$

with

$$f_i(\zeta) = \mathbb{E} \left[\frac{|\hat{H}_i|^2 P_i}{\sigma^2 + \sum_{j=1}^2 [\tilde{V}_j P_j K_j(\zeta) \Xi_j + (|\hat{H}_j|^2 + \tilde{V}_j) P_j \bar{K}_j(\zeta)]} \right]$$

where Ξ_1 and Ξ_2 are two independent unit-mean exponentially distributed random variables, and $\bar{K}_i(\zeta) \triangleq 1 - K_i(\zeta)$. We shall denote this infinite-layering limit (15) as $J_i^\infty(\mathbf{K})$.

Proof outline: The proof is a generalization of the proof of [3, Theorem 4] and hinges on similar ideas. The main difference is that, in the single-user setting in [3], the achievable rate converges to an expression that does not

depend on the layering function. This allows for a simplified analysis where L -variate power allocations are approximated by N -variate (for N sufficiently large) equi-power allocations

$$P_1 = \dots = P_N = \frac{P}{N}. \quad (16)$$

In contrast, for the fading MAC, the infinite-layering limit (15) depends on the pair \mathbf{K} of layering functions, so a refined analysis is required. ■

Note that [3, Theorem 4] follows from Theorem 1 by setting $P_2 = 0$ and by the change of variable $\xi = K_1(\zeta)$.

B. Vertices of the rate region

By a change of variable applied to the integral on the RHS of (15), it can be shown that $J_i^\infty(\mathbf{K})$ can be written as

$$J_i^\infty(K_1, K_2) = J_i^\infty(\tilde{K}_1, \tilde{K}_2). \quad (17)$$

where

$$\tilde{K}_1(\zeta) \triangleq \zeta + A(\zeta), \quad \zeta \in [0; 1] \quad (18a)$$

$$\tilde{K}_2(\zeta) \triangleq \zeta - A(\zeta), \quad \zeta \in [0; 1] \quad (18b)$$

for some function $A: [0; 1] \rightarrow [-\frac{1}{2}; \frac{1}{2}]$ satisfying

$$A(0) = A(1) = 0 \quad (19a)$$

and

$$\sup_{0 \leq \zeta_1 < \zeta_2 \leq 1} \frac{|A(\zeta_2) - A(\zeta_1)|}{\zeta_2 - \zeta_1} \leq 1. \quad (19b)$$

This allows us to write $J_i^\infty(\mathbf{K})$ as a functional of one function A instead of two $(K_1$ and $K_2)$, i.e., $J_i^\infty(\mathbf{K}) = J_i^\infty(A)$.

Definition 2. A function $A: [0; 1] \rightarrow [-\frac{1}{2}; \frac{1}{2}]$ with border values $A(0) = A(1) = 0$ satisfying the Lipschitz condition

$$\sup_{0 \leq \zeta_1 < \zeta_2 \leq 1} \frac{|A(\zeta_2) - A(\zeta_1)|}{\zeta_2 - \zeta_1} \leq 1 \quad (20)$$

is called a *relative layering function*. The set of relative layering functions is denoted as \mathcal{L} .

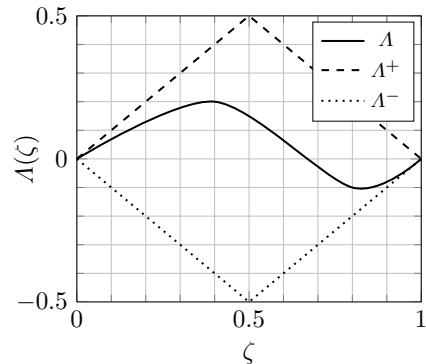


Fig. 1. Example of a relative layering function.

Figure 1 shows three examples of relative layering functions. The relative layering function A has the following

interpretation: having decoded a proportion $\zeta \in [0; 1]$ of the overall signal power $P_1 + P_2$, the value $2\Lambda(\zeta)$ quantifies the power by which the first user precedes (or lags behind, if $\Lambda(\zeta)$ is negative) the second user.

Writing the infinite-layering limit as a function of a relative layering function allows us to establish the following monotonicity result, which will be used later to determine the vertices of the best achievable-rate region $\underline{\mathcal{J}}$, maximized over all number of layers and over all possible layering functions.

Theorem 2. *Let the relative layering functions Λ and $\tilde{\Lambda}$ satisfy*

$$\Lambda(\zeta) \leq \tilde{\Lambda}(\zeta), \quad 0 \leq \zeta \leq 1 \quad (21)$$

with the inequality being strict for at least one $0 \leq \zeta \leq 1$. Then

$$\begin{aligned} J_2^\infty(\Lambda) &\leq J_2^\infty(\tilde{\Lambda}) \\ J_1^\infty(\Lambda) &\geq J_1^\infty(\tilde{\Lambda}). \end{aligned} \quad (22)$$

Proof outline: Using a convexity argument, it can be shown that there exists a partial ordering for the layering functions according to which $K_1(\zeta) \leq \tilde{K}_1(\zeta)$ for all $0 \leq \zeta \leq 1$ implies $\underline{J}_1(K_1, K_2) \geq \underline{J}_1(\tilde{K}_1, K_2)$ and $\underline{J}_2(K_1, K_2) \leq \underline{J}_2(\tilde{K}_1, K_2)$. By an appropriate transformation (using variable substitutions in the Lebesgue-Stieltjes integral), the property is carried over to $J_i^\infty(\Lambda)$, $i = 1, 2$, yielding (22). ■

Theorem 2 suggests that successive decoding penalizes users decoded first, while it benefits users decoded last.

A direct implication of Theorem 2 is that the vertices of the rate region $\underline{\mathcal{J}}$ are obtained for the extremal functions

$$\Lambda^+(\zeta) \triangleq \begin{cases} \zeta & \text{for } 0 \leq \zeta \leq \frac{1}{2} \\ 1 - \zeta & \text{for } \frac{1}{2} \leq \zeta \leq 1 \end{cases} \quad (23)$$

and $\Lambda^-(\zeta) = -\Lambda^+(\zeta)$, $0 \leq \zeta \leq 1$.

Corollary 1. *The relative layering functions Λ^+ and Λ^- satisfy*

$$J_2(\Lambda^+) = \sup_{\Lambda \in \mathcal{L}} J_2^\infty(\Lambda) \quad (24a)$$

$$J_1(\Lambda^-) = \sup_{\Lambda \in \mathcal{L}} J_1^\infty(\Lambda). \quad (24b)$$

While Theorem 2 provides an easy handle on the vertices of $\underline{\mathcal{J}}$, it is difficult to investigate the set of points in $\underline{\mathcal{J}}$ of maximal sum rate that are not vertices. To better understand the behavior of these points, we define four families of relative layering functions $A_{k,\alpha}$, $k = 1, \dots, 4$ parametrized by a scalar α where $A_{k,\alpha}$ is continuous in α and the extremal functions Λ^+ and Λ^- are contained in each family. By varying α , we can move from one vertex point to the other. It is unknown whether any of these functions achieves the maximal sum rate for α 's for which $A_{k,\alpha}$ is neither Λ^+ nor Λ^- .

We define $A_{k,\alpha}$ as integrals $A_{k,\alpha}(\zeta) = \int_0^\zeta A'_{k,\alpha}(z) dz$ over their respective derivatives:

$$A'_{1,\alpha}(z) = \alpha \left(\mathbb{I}_{[0;1/2]}(z) - \mathbb{I}_{[1/2;1]}(z) \right), \quad \alpha \in [-1; 1]$$

$$A'_{2,\alpha}(z) = \mathbb{I}_{[0;\alpha \cup [\alpha+(1/2);1]}(z) - \mathbb{I}_{[\alpha;\alpha+(1/2)]}(z), \quad \alpha \in [0; \frac{1}{2}]$$

$$A'_{3,\alpha}(z) = \text{sgn}(\alpha) \left(\mathbb{I}_{[0;|\alpha|]}(z) - \mathbb{I}_{[1-|\alpha|;1]}(z) \right), \quad \alpha \in [-\frac{1}{2}; \frac{1}{2}]$$

$$A'_{4,\alpha}(z) = \mathbb{I}_{[0;\alpha \cup [1/2;1-\alpha]}(z) - \mathbb{I}_{[\alpha;1/2 \cup [1-\alpha;1]}(z), \quad \alpha \in [0; \frac{1}{2}].$$

Here, $\text{sgn}(\cdot)$ denotes the sign function and $\mathbb{I}_{\mathcal{A}}$ denotes the indicator function of the set \mathcal{A} .

Figure 2 shows the sum rates achieved by $A_{k,\alpha}$, $k = 1, \dots, 4$ for a symmetric fading MAC with parameters $P_1 = P_2 = 10$ and $\sigma^2 = 1$, plotted against R_1 . The channel components \tilde{H}_i , $i = 1, 2$ are both zero-mean, circularly-symmetric, complex Gaussian random variables with variance $\frac{1}{2}$. Moreover, we choose $\tilde{V}_1 = \tilde{V}_2 = \frac{1}{2}$. Observe that the sum rate critically depends on the chosen rate-splitting scheme. Further observe that $A_{2,\alpha}$ does not achieve its largest sum rate at a vertex point. Consequently, there exist rate pairs that cannot be achieved by time sharing between the vertices.

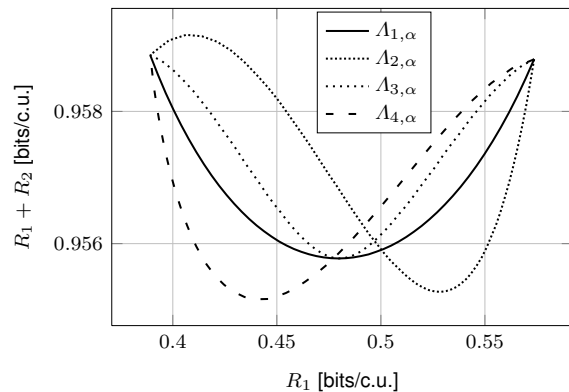


Fig. 2. Comparison of different relative layering functions.

IV. CONCLUSION

We have blended the rate-splitting approaches by [3] and [4] in order to derive a novel inner bound on the capacity region of the fading MAC with imperfect receiver CSI. We have shown that, for every pair of layering functions \mathbf{K} , the supremum of this inner bound is approached as the number of layers tends to infinity, and have derived an integral expression for this supremum. In addition, we have determined the vertices of the best inner bound, maximized over the number of layers and all layering functions. Our analysis has revealed that, in contrast to the setting with *perfect* receiver CSI, certain rate pairs cannot be achieved by time sharing between the vertices.

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